12-29-2023

Corporate Finance

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Touro University

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OPEN TOURO
NEW YORK, NY

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About the Author

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Dr. Bigel was formerly a fixed income analyst in the International Banking Department of the Bankers Trust Company (now DeutscheBank), analyzing international wholesale loans and debt instruments, and a graduate of its Institutional Credit Training Program. He later was affiliated with the Ford Motor Company, conducting investment analysis and planned car profits analysis, annual budgeting, and strategic planning. Subsequently, he worked as a senior portfolio manager attached to the wealth management division of Prudential Securities. He was formerly registered under Series 3, 7, 15, 24, 63, and 65.

As an independent consultant, he was involved in numerous high-profile cases including Enron. Dr. Bigel has

conducted executive education programs for Morgan Stanley Capital Markets, Merrill Lynch Capital Markets, UBS, Lehman Brothers, CIBC, G.X. Clarke & Co. (now part of Goldman Sachs), and China CITIC Bank. He currently serves on the Financial Industry Regulatory Association's Board of Arbitrators.

His extensive published research relates to Financial Ethics and Moral Development, Behavioral Finance, and Political Economy. He has been teaching college and graduate level finance courses since 1989.

Dr. Bigel has been interviewed on American radio, was a visiting scholar at Sichuan University and at Xi'an Jiaotong University in China, and appeared on Chinese television. At Touro University, he is a member of the Faculty Senate, The Touro Academy of Leadership and Management, The Assessments Committee, and The Promotions Committee. He chairs the Integrity Committee at LCM.

His wife and their three children reside in New York City. He enjoys reading, playing 60's guitar, seeing his students succeed professionally, and watching his kids grow.

Educational Background:

- Ph.D., (high honors) New York University, Steinhardt School of Culture, Education, and Human Development (Business Education and Financial Ethics)
- M.B.A., New York University, Stern School of Business (Finance)
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- CFP™, International Board of Standards and Practices for Certified Financial Planners
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Acknowledgements

Open Touro wishes to thank the following individuals who served as blind reviewers for *Corporate Finance*, providing invaluable feedback toward its improvement during the development process:

Darleen Braunshweiger J.D.
Nassau Community College

Henry Dachowitz, CPA, MBA
Stern School of Business, New York University

Larry Huckins, PhD
Department of Economics and Finance
Baruch College, City University of New York

Amy Jansen
Business Research Librarian
Southern Connecticut State University

Guy P. Lander, Esq., J.D.
Member, Carter Ledyard & Milburn LLP

Dr. Kathleen Locklear, D. Mgt., MBA
Risk Management & Insurance
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Professor Raúl Fernández-Calienes, M.B.A., Ph.D.
Gus Machado College of Business, St. Thomas University

Chapter 0: Review of The Time Value of Money

0.1 Chapter Zero Learning Outcomes

Learning Outcomes

- **Calculate** and **Apply** simple Present and Future Values, Ordinary Annuities, and Perpetuities in specific contexts.
- **Distinguish** between Internal and External funds.
- **Explore** the changes in a firm's financial ratios over time.

0.2 The Time Value of Money and Interest

For each of the following questions, assume you have $1 and that interest on it will be paid once, at the END of the stated period. What is the future value of each of the following? (If you had more than $1, the answer would be the appropriate multiple thereof.)

In other words, if given one dollar, how much money would you have if, as in question #1, you earn 5% for one year, and the interest is paid to you at the end of one year?

Try to create a symbolic formula, which may be employed, for any similar problem.

1. 5% interest, paid once a year, at the END of the year, for one year.
2. Same as above, but at 10%.
3. Same as above, but for two years.
4. 10% interest, twice a year, for one year.
5. Same as above, but for two years.
6. What happens to future values as interest rates (“R”), the number of years (“n”), and compounding frequency (“p”) increase?
7. For each of the above questions, what would be the *present value* of $1 to be received at the end of the stated periods?
8. Is this a realistic question to ask? How might this possibility actually happen?

Question #6: The Three Commandments of TVM

If Interest Rates Rise	Future Value	Present Value
Annual Interest Rate (R)	Rise	Decline
Number of Years (N)	Rise	Decline
Compounding Periods (P)	Rise	Decline

The opposite will occur if R, N, and / or P declines.

An investment in knowledge pays the best interest.

-Benjamin Franklin

0.3 The Time Value of Money and Interest (Solutions and Explanations)

The $1 in the question is referred to as "Present Value (PV)." The amount of money we will have in the future is referred to as "Future Value (FV)." Remember, we are, so far, assuming that the interest payments are made at the *end* of each relevant payment period.

Question	Calculation	What we learn from this
1	$1 (1.05) = $1.05	$1 times one (representing 100% of your principal) plus the interest rate
2	$1 (1.10) = $1.10	As *interest rates* increase, so too do FVs.
3	$1 (1.10) (1.10) = $1 $(1.10)^2$ = $1.21	As *time* increases, so too do FVs.
4	$1 (1.05) (1.05) = $1 $(1.05)^2$ = $1.1025	Rates are always quoted in annualized terms, unless otherwise indicated. You must make the necessary adjustments. Here we see that as the number of compounding periods per year increases, so too does the FV.
5	$1 $(1.05)^4$ = $1.2155	We have two things going on here
6		See "summary" below
7.1	$1 ÷ 1.05 = $0.9524	Whereas solving for FV involves multiplication (i.e., compounding), solving for PV involves division or discounting.
7.2	$1 ÷ 1.10 = $0.9091	As interest rates increase, PVs decrease
7.3	$1 ÷ $(1.10)^2$ = $0.8264	As time increases, PVs decrease
7.4	$1 ÷ $(1.05)^2$ = $0.9070	As discounting frequency increases, PV decreases
7.5	$1 ÷ $(1.05)^4$ = $0.8227	
8.	Yes!	This is how bonds and many financial things are figured. Can you provide examples?

Definition: A "reciprocal" is the "opposite" of a number, which is arrived at by turning the number on its head – by dividing! So, 1/2 or 0.5 is the reciprocal of 2. The reciprocal of 5 is 20% (i.e., 1/5).

Summary: As interest rates, the number of compounding periods per year, and time increase, the Future Value increases and the Present Value decreases.

0.4 Ordinary Annuities

An annuity is a series of cash flows that must satisfy both of the following conditions, in order to qualify as an annuity, namely it:

- Arrives (or leaves) in *regular* intervals
- Consists of *equal* dollar amounts

If a series of cash flows may be defined as an annuity, then a short-cut mathematical or tabular method may be used to figure the series' FV/PV.

In Investments, most (if not all) annuities may be qualified as "Ordinary Annuities," since their cash flows occur at the end of the relevant periods.

Other annuities are called "Annuities Due," meaning that the CFs occur at the start of the relevant period.

If the cash flows do not qualify as an annuity, then its FV/PV may be derived only by calculating the FV/PV of each discrete CF and then aggregating.

This is the same process by which we shall derive the "short-cut" Annuity TVM factors.

0.5 The Derivation of Ordinary Annuity Factors

You are given the following information. Column by column, complete the table by filling in the appropriate future value factors ("FVF"), the future values of each respective cash flow ("FVCF"), as well as the same for the present value factors and cash flows ("PVF" and "PVCF"). Once completed, add up the columns at the bottom.

Given:

3-year annuity

$100 received per year

Annual Discounting/Compounding

Factor = R = .10

Period	Cash Flow	FVF	FVCF	PVF	PVCF
1	$100				
2	$100				
3	$100				
Annuity Factors					
Dollar Values					

Note that here we are dealing with "ordinary" annuities, which means that all the cash flows in the series are received at the *end* of the relevant period. Soon we will examine another convention.

0.6 The Derivation of Annuity Factors (Solution)

Below you will find the solution to the problem on the prior page. Note that if you had an interest rate table for annuities, you would be able to multiply the annuity cash flow (in this case, $100) by the appropriate factor and arrive at the future- or present-values of the cash flows, in one step. Such annuity interest tables exist (and may be found at the back of this text, together with the tables for "simple" future- and present- value factors).

Period	Cash Flow	FVF	FVCF	PVF	PVCF
1	$100	*1.2100*	*$121.00*	*.9091*	*$90.91*
2	$100	*1.1000*	*$110.00*	*.8264*	*$82.64*
3	$100	*1.0000*	*$100.00*	*.7513*	*$75.13*
Annuity Factors		*3.3100*		*2.4868*	
Dollar Values	*$300*		*$331.00*		*$248.68*

What may we observe from this table?

Future value annuity factors are always greater than the number of periods. Here the FVF was 3.31, or greater than $n \cdot p = 3$ periods. This is because the annuity multiplier is the sum of each respective yearly factor, each of which is greater than 1.0, since they are all multiples of $(1 + R)^n$. (This assumes that $R > 0$.)

Contrarily, each PVF is less than the number of periods because the respective factors are all less than 1.0, as each factor is the reciprocal of $(1 + R)^n$. (Again, this assumes that $R > 0$.)

When utilizing a table, it is always a good idea to "eyeball" the factors you are using to make sure you didn't lift the figure from the wrong table or make some other error. Use your head at all times; do not be a robot!

The present value of the annuity is $248.68. If you, alternatively, had had a single sum in the amount of $248.68, and had invested it for three years at 10%, you would have $331 at the horizon:

$$(\$248.68)\ (1.10)^3 = \$331$$

0.7 Future and Present Annuity Factors: Mathematical Formulas

Let's "put on the table" the formal mathematical formulas for ordinary annuities' factors. Remember: a "factor" is a multiplier (for the given cash flows). These formulae will be useful when your tables do not have a certain interest rate that you need, and especially when you need to calculate a fractional rate, e.g., 10.23%.

Key: PVAF: Present Value Annuity Factor AND FVAF: Future Value Annuity Factor

Present Annuity Factors:

$$\mathbf{PVAF} = [(1) \div (R/p)] - [(1) \div (R/p)\,(1 + R/p)^{n \times p}] \quad \mathbf{Or}$$

$$\mathbf{PVAF} = [(1) \div (R/p)] - \frac{1}{(R/p)\,(1 + R/p)^{n \times p}}$$

Example 1: R = 0.10; N = 5; P = 2

Solution 1: $[(1) \div (0.10/2)] - [(1) \div (0.10/2)\,(1 + 0.10/2)^{5 \times 2}] = 7.72173493$

*This multiplier should be the same as in your Present Value Annuity Table

Example 2: R = 0.095 N = 5 P = 2

Solution 2: $[(1) \div (0.095/2)] - [(1) \div (0.095/2)\,(1 + 0.095/2)^{5 \times 2}]$ = Fill in your answer

*This multiplier is not in your Present Value Annuity Table. Compare the two answers.

Future Annuity Factors:

$$\mathbf{FVAF} = [(1 + R/p)^{n \times p} - 1] \text{ ¸ } R/p$$

Example 3: R = 0.10; N = 5; P = 2

Solution 3: $[(1 + 0.10/2)^{5 \times 2} - 1] \div 0.10/2 = 12.57789253554883$

*This multiplier should be the same as in your Future Value Annuity Table

Example 4: R = 0.1012; N = 5; P = 2

Solution 4: $[(1 + 0.1012/2)^{5 \times 2} - 1] \div 0.1012/2 =$ Fill in your answer

*This multiplier is not in your Future Value Annuity Table. Compare the two answers.

0.8 Loans: The Conventional Mortgage

Mortgages are different than ordinary loans. In most loans, interest is paid over the term or life of the loan, and the entire principal is paid in one fell swoop at the loan's term or maturity. Most mortgages are self-amortizing, which means that all payments include portions of both interest and principal, resulting in decreasing principal balances over time until, at maturity, the entire loan will have been paid off. Let us see, by way of example, how this may work.

Given:	Principal:	$100,000	Rate:	9%
	Term:	10 years	Period:	Yearly

Mathematical Rationale:

The loan proceeds, i.e., $100,000 in this case, represent the present value. The "periodic payment" represents the annuity payments to be made over future years. The present value of the annuity payments should equal the loan principal:

Principal = Periodic Payment x Present Value Annuity Factor

Calculation:

Periodic Payment = Principal/PV Annuity Factor

$100,000 ÷ 6.42 = $ 15,576.32

Interest = Opening Balance x Rate

Principal Payment = Periodic ("Total Cash") Payment less Interest

Balance = Opening Balance less Principal Payment

Payment and Amortization Schedule:

	Cash Payments			
Year	**Total Payments**	**Interest**	**Amortization**	**Balance**
0				$100,000
1	$15,576	$9,000	$6,576	$93,424
2	15,576	8,408	7,168	86,256
3	15,576	7,763	7,813	78,443
4	15,576	7,060	8,516	69,927
5	15,576	6,293	9,283	60,644
6	15,576	5,458	10,118	50,526
7	15,576	4,547	11,029	39,497
8	15,576	3,555	12,021	27,476
9	15,576	2,473	13,103	14,373
10	15,576	1,294	14,282	0
Totals	$155,763	$55,763	$100,000	

The 9% interest payments are charged against the balance of the loan. In the first year, the interest portion of the payment is 9% of $100,000 or $9,000. This leaves $15,576 less $9,000 = $6,576 going toward principal reduction, i.e., *amortization*. The new, *amortized balance* is hence $100,000 less $6,576 = $93,424. This continues for each period until maturity.

While you may be thinking in terms of borrowing, investors may invest their money in mortgage-backed securities, e.g., bonds issued by the *Government National Mortgage Association* (or Ginnie Mae). These investors are, in effect, lenders, the other side, as it were, of the same coin. The math is the same.

Note:

Mortgage payments are usually made in MONTHLY installments, and often with greater maturities. This has been simplified for illustration purposes, so that the reader may easily refer to standard interest rate tables.

The mortgage formula is important to master as it will be used again in three additional contexts: 1. Leasing; 2. Bond Accounting; and 3. Capital Budgeting: The Annual Annuity Approach.

0.9 Growth Perpetuities and the Dividend Discount Model

A "growth" perpetuity is a perpetual cash flow stream ("CF") that grows at a *constant* rate of growth, which we shall call "*g*." It is a special case of perpetuity. If we are to receive a cash flow of $100 and its growth rate is 5%, the next cash flow would follow this formula: (Cash flow now) (1 + G) = Cash flow in next period.

$$CF_1 = CF_0 (1 + G)$$

$$\$100 (1.05) = \$105$$

The formula for a growth-perpetuity is: (Cash flow next period) ÷ (Discount rate – Growth rate). Symbolically, this may be expressed as:

$$PV = CF_1 \div (R - g)$$

In the above example (where $I = 0.10$), if the growth rate had been 5%, the present value would be (assuming here that the *next* CF is $105, that is, [$100] [1.05] = $105:

$$\$105 \div (.10 - .05) = \$2{,}100$$

Notice that for this formula to work, "*g*" cannot equal or exceed "*R*."

This formula may also come in handy for cases of negative growth. Since "*g*" would be negative, in this case, the formula would require that one *add* the growth rate to the interest rate in order to determine the present value.

Suppose we say that the cash flows are $100 per year (i.e., starting with $100 next time) with a negative growth rate of 5% and a discount rate of 10%. What would the present value be? Once again, the answer would be:

$$\$100 \div [.10 - (-.05)] = \$100 \div .15 = \$666.67$$

Of course, there is also the possibility that $g = 0.0\%$.

The Dividend Discount Model ("DDM")

The DDM is attributed to Myron Gordon (1961) who asserted that a stock's value is determined by discounting its expected future cash flows, i.e., its perpetual dividend stream. This notion is not without some controversy. In any case, the DDM is very commonly used. It only requires substituting the Dividend in the above perpetuity formula for the cash flow.

In the instance where there is no growth, the DDM conforms well to Preferred Stock. In cases where there may be some expected growth in the dividend, the formula conforms, although not necessarily perfectly, to Common Stock.

Note also that $D_1 = D_0 (1 + G)$.

0.10 Chapter Zero Review Questions

Review Questions: Chapter Zero

In all the following chapters' study questions and problem sets, see if you can supplement the given questions with your own. Answer the given questions expansively.

1. Define Free Cash Flow.
2. What is Free Cash Flow used for?
3. You are given the following data. Calculate the simple Present Value:
    - Future Value = $110
    - Number of Years = 10
    - Compounding Periods = Quarterly
    - Compound Rate = 12%
4. Solve for the following:
    - Present Value = $750
    - Number of Years = 10
    - Discount Periods = Quarterly
    - Discount Rate = 12%
    - Future Value = ?
5. What is the Future Value of a Perpetuity?
6. What is the Present Value of a $125 Perpetuity, compounded quarterly, at an 8% annual rate?
7. What if that Perpetuity grew at an annual rate of 4%?
8. What are both the present- and future-values of a $663 ordinary annuity, given the following:
    - Rate = 8%
    - Years = 10
    - Frequency = Yearly
9. You just took out a $240,000 mortgage for ten years, with annual payments, and an annual rate of 5%. What is your annual payment obligation? Assume the payments are due at the end of the year.
10. How much interest will you have paid over the life of the mortgage as compared with the principal?
11. In general, mortgage interest payments go *up/down*, while principal payments go *up/down*. Which are the right (italicized) choices?
12. Why take out a longer-term mortgage when you can afford a shorter-term one with a higher payment?

Chapter 1: Capital Budgeting

1.1 Chapter One Learning Outcomes

Learning Outcomes

- **Explain** the concept of *Free Cash Flow*.
- **Import** *Free Cash Flow* data into the Capital Budgeting decision-making process.
- **Distinguish** between Mutually Exclusive and Independent projects.
- **Calculate** all the various Capital Budgeting methods.
- **Consider** the strengths and weaknesses of each method.
- **Choose** that method which best suits a particular context.
- **Introduce** a qualitative rationale for project choice given differences in competing projects' scales.
- **Implement** a Quadratic Solution where applicable.

1.2 Corporate Financial Management: The Issues

The corporation, and by assignment, the corporate financial manager, are responsible for establishing the strategic (i.e., long-term) financial direction of the company. This involves numerous tasks.

The corporation must decide how it shall achieve growth in order to provide an increasing financial return – or profit – to its shareholders. In order to grow, the firm must raise capital, or financial resources ("money"), and use those resources in a productive, profitable manner. The firm raises capital in order to invest in assets that will produce products for sale and profit. In the process of *capital budgeting*, the manager examines the prospective investment projects available to the firm and chooses the most promising from amongst this set. This requires an evaluation of all relevant costs and resulting profits. This is a complex process.

To the extent that the corporation may not generate sufficient funds to meet all its investment needs on its own, it will have to rely on outsiders to provide "external" financing. This often constitutes the bulk of its sources of funds. Some of these funds will come from borrowing. Some of the funds will come from equity.

The proportion of how much capital shall be sourced from debt and how much from equity is a matter of the firm's *capital structure* policies. The company will attempt both to minimize its overall cost of capital, while, at the same time, to minimize its financial risks. Some capital sources present greater financial risks to the corporation than others.

In addition to managing its long-term financial affairs, the company must manage its short-term financial affairs as well. This includes the profitable handling of its cash, inventory, accounts receivable, and other critical short-term assets. Each of these areas requires expert care.

To summarize, corporate financial management is all about capital budgeting, capital structure, and short-term financial management. The objectives are to provide profits, minimize risk, and thereby enhance shareholders' wealth by maximizing the market value of the corporation's shares (and perhaps by providing dividends). We will cover Capital Budgeting first. Here is the Capital Budgeting process:

Capital Budgeting Stages:

1. Find potential investment projects.
2. Estimate (i.e., project) Free Cash Flows ("FCF" is discussed immediately below).
3. Evaluate the investment project candidates using Capital Budgeting techniques.
4. Implement and monitor the chosen project.

1.3 Free Cash Flow

A corporation's value is dependent, in large part, on the income and cash flow it produces. Cash flow is different from income in that "income" is based on "accrual accounting," which is transaction-based, and will reflect certain non-cash events such as depreciation, and other idiosyncrasies. One key idiosyncrasy has to do with "timing" – the accountant may book a sale, for example when "constructively received," i.e., when legally and economically receivable, but not yet received in actual cash.

Under accrual accounting, the accountant records economic transactions rather than the movement of actual cash received and paid out; this results in "timing" differences between the two approaches. (In theory, in the long run, such accounting vs. cash differences even out.) Because of these differences, financial analysts who are more cash-oriented must make certain adjustments to the accounting data in their own calculations and projections.

In a certain sense, cash flow is more important to corporate valuation than income because dividends are (usually) paid out to investors in cash. Further, when engaging in corporate planning, a possible capital investment will be judged attractive dependent upon the cash flow it produces in the future because it is cash that, arguably, fuels growth.

Capital investments, i.e., "growth" investments, include expenditures for hard assets, as well as for product development, and much more. "Free cash flow" refers to funds that a project, or the corporation as a whole, generates beyond its own internal and ongoing needs. We may think of free cash flow as the cash which is left over from an investment project, after all net operating funds generated by the project are utilized for necessary, non-discretionary purposes, including hard assets' maintenance and replacement, and increases in working capital; this "left-over" amount may be used for other financial purposes, such as fixed asset expansion, at the discretion of management.

The purpose of projecting the firm's FCF is to get a handle on the firm's future growth prospects after paying for non-discretionary capital and other expenditures. The more FCF the firm generates, the greater the firm's growth prospects, the greater its management discretionary alternatives for choosing still further growth-oriented investment projects. It is, after all, growth which drives share price appreciation. We assume that the firm has a never-ending appetite for growth.

As it is true for the corporation, so too would it also be true for any individual capital investment project. The corporation will favor any specific investment project that provides maximum FCF.

What can the corporation do with its FCF?

- Purchase more P, P , & E, and expand its inventory
- Invest in Mergers and Acquisitions (using shares when high-priced)
- Increase Research & Development
- Pay discretionary (common stock) dividends
- Buy back common shares (when the shares are cheap)
- Pay off Debt

Once again, a project that throws off "a lot" of FCF is desirable. A firm, as a whole, that produces lots of FCF

may be thought of in a most positive light – as one, among other possibilities, that has a lot of growth potential and makes for a good investment.

For information regarding the calculation of Free Cash Flow, please refer to the *Introduction to Financial Analysis* text by this author.

1.4 Capital Budgeting: The Investment Decision

The phrase “Capital Budgeting” begs explanation. This phrase obviously consists of two terms. Let’s take “budgeting” first. Budgeting refers to the process whereby limited financial resources are allocated to alternative investment choices. If an individual has only $25,000 to spend on an automobile due to the limits of her budget, that Ferrari is not going to happen.

> *You’re right. I did lose a million dollars last year. I expect to lose a million dollars this year. I expect to lose a million dollars next year. You know, Mr. Thatcher, at the rate of a million dollars a year, I’ll have to close this place in… 60 years.*
>
> – Orson Welles, *Citizen Kane.*

If your funds are unlimited, why budget? No one has unlimited funds (except Citizen Kane!). Thus, one must decide what is the best choice of investment with only limited funds. Budgets can be applied to spending on short-term resources, such as food in an individual’s case, or on inventory or wages in the case of a corporation. In any event, such “operating” expenditures are consumed during the accounting period, and may be regulated by the annual “operating budget” established corporately.

Corporations occasionally spend money on “capital” assets, i.e., long-term needs as well. These investments (the word “investment” itself connotes long-term) are not consumed during the accounting period, but over many future periods. Indeed, the *External Funds Needed* (“EFN”) formula projects the amount the corporation will need in order to acquire additional assets to fund growth. The process by which capital resources are allocated to long-term investments is referred to as “Capital Budgeting” (CB). The corporation must make optimal investment decisions given its limited capital resources. “Optimal” investment decisions are those which maximize return (given an acceptable risk level), profits, and firm wealth.

On the following pages we will examine numerous mathematical and analytic techniques for capital budgeting. We shall assume that an analyst, ex-ante, has provided pro-forma accounting income, (free-) cash flow, or similar data, which shall then be subjected to Capital Budgeting analysis. We shall further (unrealistically) assume that the forward-looking data are 100% certain as to their actual future occurrence. Finally, we will not evaluate qualitative considerations, which may alert the analyst, in practice, to recommend some other strategy or alternative, even when the numbers may “speak” otherwise. In reality, the analyst will have to “know the business” in order to engage in such departures.

Investment-, or capital-projects, may be categorized as either “independent” or as “mutually exclusive.” This categorization will impact the capital budgeting decision, as discussed below.

1.5 Independent versus Mutually Exclusive Projects

In making the evaluation of whether to accept or reject a proposed project, the analyst must keep in mind two matters:

- Whether the projects are *independent* or *mutually exclusive*
- The relevant decision rules, or "rules of thumb," for accepting or rejecting a project

Independent Projects – If there are, say, just two projects under consideration, both, just one, or none of the projects may be accepted based on their separate, individual merits using the rules of thumb to be described below. The acceptance of one project has no bearing on the acceptance or rejection of another. There is both enough physical space and/or financial resources for both projects, if warranted.

Mutually Exclusive Projects – If there are, say, just two (competing) projects under consideration, just one or none of the projects may be accepted based on their relative merits. The acceptance of one project precludes the acceptance of the other. There may not be enough physical space and/or capital funds for both investment projects. If neither project meets minimal "metrics" (i.e., "rules of thumb") no project may be accepted.

Imagine two potential investment projects, "A" and "B," as below. The question is whether one may accept "A" or "B" or some other combination.

Independent	**Mutually Exclusive**
A & B	Not both
A only	A only
B only	B only
Neither	Neither

If the rule of thumb demands it, whether the projects are independent or mutually exclusive, you may reject both projects. You may only accept both if the projects are independent. If mutually exclusive, you may not be able to build two factories in the same space. Perhaps there will not be enough funds for both.

1.6 The Payback and Discounted Payback Methods

The analyst's job is to assess the financial merits of a potential capital project based on Free Cash Flow projections, and to make a recommendation to either "accept" or "reject" it accordingly. Presented below are two basic analytic approaches, presented as a starting point for project investment analysis, i.e., Capital Budgeting.

Code:

PVF: Present Value Factor

PVCF: Present (discounted) Value of the Cash Flow

Analyst's Pro-forma		**Payback Method**	**Discounted Payback Method**		
Period	**Free Cash Flow**	**Cumulative Cash Flow**	**PVF** (r =.10)	**PVCF**	**Cumulative PVCF**
0	($10,000)	($10,000)	1.0000	($10,000)	($10,000)
1	1,000	(9,000)	0.9091	910	(9,090)
2	2,000	(7,000)	0.8264	1,653	(7,437)
3	3,000	(4,000)	0.7513	2,254	(5,183)
4	4,000	000	0.68308	2,732	(2,451)
5	4,000	4,000	0.6209	2,484	33
6	4,000		0.5654	2,262	2,295
7	4,000		0.5132	2,053	
8	4,000				
n	4,000				

You will note above that in "year 0," the firm invested $10,000 in the project. In the first year, the project is expected to produce $1,000 in free cash flow, so that the firm will then be "behind" by only $9,000. This goes on until the project recovers its initial cost, and thereafter produces "profits," in some sense.

The discounted payback adjusts the cash inflows for the time value of money. This adjustment reduces the nominal values to their respective present values, and therefore, extends the length of the payback. We have used a discount rate of 10% above.

The decision rule for the payback identifies a *preference* for the project, among competing alternatives, with the shorter (discounted) payback – here only one project is presented, but you may imagine another with a different payback. The simple payback method does not account for the time value of money, and hence is analytically deficient. In our example, the proposed project's payback is four years, and the discounted payback is just over

four. Take note that Payback is expressed in terms of years. Should another competing project have a shorter (discounted) payback, the analyst would *prefer*that.

The discounted payback indeed accounts for TVM, but is deficient in that it ignores any cash flows after the payback period. Imagine you have two competing, mutually-exclusive projects and you choose the one with the shorter discounted payback. The rejected project however, may have substantial, and vastly superior, cash inflows to be received after its payback. The method (but perhaps not you!) ignores these later cash flows, which may contribute substantially to the firm's financial position and profits.

The payback methods do not provide any rule of thumb in the matter of the evaluation of a single, independent, non-competing project. What does it mean – financially – when we may say that a project's payback is "four" years? Shall we accept it? On what objective basis shall we do so?

Question:

What if the *sequence* of the first four cash inflows were reversed? Under each method, what would the Paybacks be?

Answers:

1. There would be no change in the case of the simple Payback.
2. In the case of the Discounted Payback, the greater cash flows would come in sooner and the payback would be shorter, due to TVM!

Assumptions for "Ranking Criteria":

Earlier, we used the word "Preference." In other words, we could rank-order our project preferences using the relative Payback (or Discounted Payback) periods of competing projects. Projects with shorter paybacks would be preferred, but not necessarily accepted. These methods provide no "Rule of Thumb" which informs us explicitly whether to accept ("green light") or reject ("red light") the project.

We are also assuming a known Discount Rate. The discount rate used would be the corporation's "Cost of Capital." You will recall that the Free Cash Flow projections excluded the cost of (Debt and Equity) Capital. We now bring that cost back in – in the form of the discount rate. In a later chapter, we will discuss the meaning of, and formula for, calculating the Cost of Capital. For now, it will be a "given."

1.7 Personal Financial Planning Problem: Payback Method

Problem: Shlomo is 62 years old and is entitled to Social Security retirement benefits of $800 monthly until he passes on. If he waits until age 66, he would be entitled to greater benefits, in the amount of $1,200 per month. He turns to the Payback Method to decide.

Solution: If Shlomo takes the benefits now, he will receive (4) (12) ($800) = $X that he would not otherwise have had at all if he waited the four additional years. If he does wait the four years, he will receive $400 more per month. At that rate, it will take him $X / $400 = Y months, or (y / 12) = Z years, to break even.

Questions:

1. Calculate out X, Y, and Z.
2. Does Shlomo's (expected) longevity matter to his choice of action?
3. Would it matter if he did a Discounted Payback?
4. What would YOU do? What would you tell Shlomo?

Note:

Social Security retirement benefits increase each year, so that it may pay to wait, in any event. One can wait until 70 before taking the benefits.

1.8 Personal Financial Planning Problem: Payback Method (Solutions)

Answer #1

X = $38,400

Y = 96 months

Z = 8 years.

Shlomo would have to live beyond 66 + 8 = 74 years for his decision to wait four years to take the benefits to have paid off.

Answer #2

His longevity matters. If he expects to live beyond age 74, he should delay the receipt of his retirement benefits.

Answer #3

If he calculates a discounted payback, he would have to live even longer depending on the discount rate. The higher the discount rate, the longer he would need to live.

Answer #4

An advisor should learn some things about Shlomo's health and family. Is he likely to live long? Does he wish to provide for his family?

1.9 Payback and Discounted Payback Summary

The following summarizes some critical issues relative to the use of *Payback* and *Discounted Payback* methods.

- Either method provides the analyst with a preference or ranking system. Clearly one would prefer a shorter payback to a longer payback; a four-year payback is preferred to a five-year payback.
- Neither method provides you with a clear *rule of thumb*, i.e., a clear decision rule, by which one can assess whether to categorically accept or reject a project. Saying merely that the payback is "four years" alone is insufficient. Given an independent project, what would you do?
- The simple payback method is inconsistent with the time value of money. A serious implication of this shortcoming may be illustrated by two competing projects both of which have the same paybacks. If one project has relatively larger cash flows coming in sooner, its discounted payback may be shorter than the other's discounted payback.
- Neither method considers any cash flows that may come in subsequent to the payback. A project may be rejected due to its longer payback; however, it may have relatively larger cash flows coming in subsequent to the payback, which are ignored by methodology.
- We have assumed that the inputted Free Cash Flow ("FCF") projections are certain, which they are unlikely to be. This text will not deal with "Capital Budgeting Under Uncertainty" methods.
- Neither method provides a Rate of Return, which most interests Financial Analysts.

1.10 Decisions and Uncertainty

Photo by Holden Baxter on Unsplash

Perfect asepsis is not possible; that does not mean that we operate in the sewer.

-Bernard Lewis,

paraphrasing Robert Solow 1986

The only certainty is that there is no certainty. Reject absolute answers and recognize uncertainty, weigh the probabilities, don't let uncertainty paralyze you, and evaluate decisions not just on the results, but on how they're made.

– Robert Rubin,

Former CEO Citibank, Goldman Sachs

Former United States Secretary of the Treasury

167th Commencement, New York University
Main speaker, 1999
New York Times (05.14.99)

1.11 Critical Methodological Issues Relative to Choice of Capital Budgeting Technique

On the pages to follow, we will present and evaluate various analytic capital budgeting techniques; so far, we have only presented and critiqued the (two) payback methods. In considering the method by which the analyst will evaluate a proposed capital investment project, s/he needs to consider whether the technique employed properly considers each of the following relevant analytic considerations, which are presented once again in review.

The Time Value of Money– Does the method properly utilize TVM? We know that nominal cash flows cannot be evaluated on an even plane!

Cash flows (or profits) after payback – Does the method consider *all* possible, projected cash flows? It is possible that one may choose a four-year payback versus a five-year alternative, but the five-year possibility provides substantially more cash inflows after the payback? This could lead to a poor choice.

Scale of the Project – Even though one project may be preferred to another in terms of, say, its discounted payback, a one-million-dollar investment with a longer payback may be preferred to a one-thousand-dollar project with a shorter payback. The larger project, simply put, may provide greater profits and/or cash flow. This issue was not discussed above.

Rate of Return (ROR) – in Finance, we typically discuss or evaluate investments in terms of an ROR; that is our language. For example, if I am considering putting money in the bank, the banker will provide me with a quote stated in percentage terms. Percentages provide a manner in which alternatives may be objectively compared.

Risk – The principal risk is that the actual cash flows don't come in as projected. The FCF projections are uncertain. In fact, the more distant cash flows are less certain than the earlier ones. To deal with this contingency, the analyst may use probability or sensitivity analysis to give some "color" to the projections – prior to subjecting the projections to a capital budgeting test. We will not deal with that (advanced) approach herewith and instead will view the projections as though they are certain.

1.12 Net Present Value (NPV)

Definition of NPV: The NPV provides the analyst with the present value, after deducting costs, of the project's pro-forma cash flows (or accounting income) in one consolidated, TVM-consistent, number. It tells the analyst the extent to which the project is expected to enrich the firm – or not – on a "present value" basis.

An example follows. See if you can complete the spreadsheet, given the assumptions below.

Example 1: "Uneven and Growing Cash Inflows"

Given:

1. Initial Outlay: $2,500

2. Cash Inflows (CF): As below

3. No salvage value ("Salvage Value" is the residual value of equipment, i.e., after it has been used up. For example, machinery may be sold for its scrap value at its life's end.)

4. $k = 8\%$ (i.e., the discount rate or "cost of capital")

Year	Free Cash Flow
1	$250
2	500
3	1,000
4	1,500
5	2,000

Solution: Fill in the appropriate data below.

Year	Free Cash Flow	PV Factor	PV of the Cash Flow
0			
1			
2			
3			
4			
5			
		NPV =	

1.13 Net Present Value (NPV) (continued)

Let's try another.

Example 2: "Uneven and Declining Cash Inflows"

Given: k = As above, except the cash inflows are reversed.

Solution: Fill in the appropriate data below.

Year	Cash Flow	PV Factor	PV of the Cash Flow
0			
1			
2			
3			
4			
5			
		NPV =	

Questions:

- Will the NPV increase or decrease relative to the prior example?
- How does the NPV method deal with each of the "critical methodological issues" outlined above?

Cost of Capital: You will recall that, in discussing the "free cash flow" formula earlier, we intentionally ignored interest expense, as it is a capital (and not an operating) cost, the latter of which we said we would use later as the discount rate. Well, here we did just that! The projections we have used herewith as a given, or starting point, have been operating cash flows, operating (accounting) profits, or Free Cash Flows (depending on who is doing the analysis) however defined. Such projections exclude capital and other non-operating costs. The analyst just "spreads" the cash flow projections based on what he understands the future business prospects of the project to be, which are then imported into one or another Capital Budgeting technique.

The discount rate used for the NPV calculation is a combined, or average, capital cost figure including debt and equity. Under the section somewhat below, we will mathematically define "cost of capital," but for now the discount rate will be treated as a given. Again, suffice it to say for the moment, that the firm's cost of capital is a kind of average of all the firm's capital costs, including debt and equity.

The NPV method has two principle deficiencies, which we – eventually – will have to resolve:

1. It does not provide us with a Rate of Return, which is more commonly used in Finance.
2. Most people, even business executives, do not understand discounted present value.

1.14 NPV Solutions

Example #1:

Year	Cash Flow	PVF	PVCF
0	($2,500)	1.0000	($2,500)
1	250	.9259	231.48
2	500	.8573	428.65
3	1,000	.7938	793.80
4	1,500	.7350	1,102.50
5	2,000	.6806	1,361.20
		NPV=	**1,417.63**

Example #2:

Year	Cash Flow	PVF	PVCF
0	($2,500)	1.0000	($2,500)
1	2,000	.9259	1,851.80
2	1,500	.8573	1,285.95
3	1,000	.7938	793.80
4	500	.7350	367.50
5	250	.6806	170.15
		NPV =	**1,969.20**

Note: that the *timing* of the cash flows is included in the NPV model.

The decision rule for NPV is to:

1. Accept any independent, non-competing project with a positive NPV. The NPV is a useful measure as it tells management by how much it may expect the project to increase the firm's wealth on a present value basis.
2. Accept that project – among mutually exclusive alternatives – whose NPV is greatest, assuming it is positive.

He who knows only his own side of the case knows little of that.

–John Stuart Mill

1.15 NPV Practice Problem

All you need to solve the following problem is in the table below. Fill in the cells.

Year	Free Cash Flow	PVF @ 6 %	PVCF
0	($1,00,000)		
1	750,000		
2	800,000		
3	810,000		
4	600,000		
5	250,000		
		NPV=	

Here's the solution. Don't look at it, when solving the problem above…

Year	Free Cash Flow	PVF @ 6 %	PVCF
0	($1,00,000)	1.0	($1,00,000)
1	750,000	0.9434	707,550
2	800,000	0.8900	712,000
3	810,000	0.8396	680,076
4	600,000	0.7921	475,260
5	250,000	0.7473	186,825
		NPV=	**$1,761,711**

Remember: The NPV is arrived at by using the company's *cost of capital* as the discount rate.

1.16 NPV Application: Trading in a Car to Save on MPGs

Yossi Kamtzan is considering buying a new car. His current car, which he purchased five years ago, is now ten years old, still in very good condition, but is a terrible gas guzzler. He is considering getting a newer model (used) car in order to save on the high cost of gasoline.

Mr. Kamtzan thinks it would be really cool to own a hybrid vehicle, but cannot find one within his targeted price points; they just have not been in production long enough, and the market for used hybrids is very small. He figures that he will keep the new car for five years before he, again, engages in another trade-in. He cannot figure what it will cost him to trade up again then, so he ignores that consideration in his calculations; he does not think it is relevant anyway. Additionally, he does not think that there will be any material differences in insurance and maintenance costs for the new car. He garages his current vehicle and takes very good care of it, and expects to do the same going forward. He has the cash and will use it to trade up, rather than borrowing.

Does it pay for him to trade in his old car for a newer one with a five-year life? Here are the relevant details.

Value of old car: $5,000

Cost of newer, used car: $20,000

Gas Mileage:

- Old car: 12 MPG
- New car: 25 MPG
- He drives 5,000 miles per year
- Cost of Gasoline: $4 per gallon

This individual may choose to either finance the newer car with cash if has the money or borrow the money if he must or wishes to. We will make a simplifying assumption that the *Opportunity Cost* of the $15,000 out-of-pocket (cash) newer car expense and the borrowing cost are the same: 6%.

(We shall assume that the borrowing cost of financing the net purchase is the same.)

Use the table below to solve this problem:

Can you imagine a corporation doing a similar (investment) analysis? Provide an example.

1.17 Trading in a Car to Save on MPGs (Solution)

To solve this problem, we first need to figure the annual gas savings of the new car and next, the present value of the calculated savings over the five anticipated years of use.

Annual gas savings	Current gas consumption (gallons): (5,000) ÷ (12) =	416.67 g.	
	Expected gas consumption: (5,000) ÷ (25) =	200.00 g.	
	Dollar value of gas savings: (216.67 g.) ($4) =		$866.68
PV of annual savings (at 6% for 5 years)	($866.68) (4.2124) =		$3,650.75

Now that we have this information, we may compare the savings over the anticipated life of the new investment, against the initial net cost of the trade-in. Again, we assume that other costs of the new vehicle are the same as the old vehicle's and, hence, do not figure any such costs into the analysis. (This, in fact, may or may not be the case.) No salvage cost or similar horizon-figures are assumed.

Initial Net Cost		($15,000)
Annual gas savings	$866.68	
PV of gas savings		$3,650.75
NPV		($11,349.25)

Judging from this analysis, it does not appear that the trade-in pays off. The NPV is negative. Do you agree with this analytic approach and decision rule? What may have been omitted?

1.18 Should I Buy a Home – or Rent? The Considerations

In considering whether it pays to invest in and own residential property or to rent a residence, here are some quantitative factors that one may insert into an NPV comparative analysis. Can you identify any qualitative factors?

Buying

Closing Costs include the down payment, title search and insurance fees, legal fees, and others.

Recurring Costs include mortgage payments (both principal and interest), condo or coop common charges including maintenance, property taxes and homeowner's insurance. Property Taxes are value-based. Additionally, the interest portion on the mortgage held by the coop or condo association is tax deductible; a mortgage on the building and paid by the coop or condo association is included, *pro rata*, in the common charges.

Opportunity Costs indicate what you might have made had you invested the down-payment and the closing costs in something else instead.
Net Sale proceeds equals cash from the sale of your home less closing costs, including the broker's commission, less the remaining balance on the mortgage, less any capital gains tax paid on profits exceeding the capital gains exclusion.

Renting

Initial Costs include the security deposit (usually equivalent to one- or two-months' rent) and, if applicable, a broker's fee.

Recurring Costs include the monthly rent and renter's insurance.

Opportunity Costs represent the alternative return one might have earned had s/he not had to pay initial costs and recurring costs.

Net Sale Proceeds include the repayment of the rental security deposit, which is returned at the end of the rental period or lease.

1.19 Net Present Value (NPV): Annuity Cash Inflows

Example #3: Annuity Cash Flows: This is a simple example – due to the cash flows' being an annuity, we do not have to use a spreadsheet table in order to discount each individual cash inflow one-by-one.

Given:

1. Initial outlay = $2,532
2. Free Cash inflow (EBT) = $1,000 per year for 5 years
3. Salvage value = $0
4. k = 10%

Solution: NPV = PV of inflows less initial outlay

= ($PV_{\text{annuity for 5yrs @ 10\%}}$ for $1,000) – ($2,532)

= $3,790.8 – 2,532

= **$1,258.80**

We will be using "Example #3" repeatedly for the next twenty pages or so, so store the "givens"!!!

You should *verify* the solution above (i.e., check it out on your own!)

If the salvage value had exceeded zero, we would have had another cash flow (for the salvage value), which would be received at the horizon. The asset would be sold at that time for its salvage value.

1.20 The Equivalent of the Multiple Cash Flows as A Singular Cash (Out-) Flow: "$1 to $2" The Rate of Return

You can readily see that indeed there exists a "zero" NPV solution for this – and any – NPV problem. That is to say that there exists a theoretical discount rate at which the project's NPV equals zero. This abstract rate is called the "Internal Rate of Return." The concept of the IRR shall occupy us more so soon.

Perhaps another, simpler manner in which example #4 may be understood better is the following. Let's not look at a *series* of cash flows, instead let's just look at *one* present- and *one* future-cash flow. Imagine you have one dollar, and that in five years it will grow to two dollars. That would imply a rate of (compound-) growth of (approximately) 15% per year. How did we get that? Solve for "R" below.

$$\$1 (1 + R)^{5} = \$2$$

$$R = .1487 (R \approx .15)$$

$$\$1 (1.1487)^{5} = \$2$$

[See "Note 1" below, if you have trouble solving for "R."]

This is the same as saying that the present value of $2 at a discount rate of 15% is $1. To be precise:

$$\$2 \div (1.1487)^{5} = \$1$$

At a discount rate of (approximately) 15%, the NPV would be zero because the cost is $1 and so is the present value of the one, $2 future cash inflow!

$$\$2 \div (1.1487)^{5} - \$1 = \$0$$

Clearly, this does not mean that you have made no money at all! Indeed, you have – 15%! You earned 15% on the dollar you invested at the start of the five-year period!

Thus, the IRR expresses a kind of rate of return (ROR) for the project; it does not mean that the NPV for the project is zero! It does not mean that you have made no money!

In contrast, the NPV provides a net dollar figure, given a pre-specified discount rate representing the firm's cost of capital; the dollar figure, which the NPV presents, indicates the expected increase in wealth to the corporation from having invested in the project – in present value terms, and after accounting for the firm's discounted, capital costs.

Question: Can you relate the NPV method to the various technical concerns discussed earlier? (We shall review this later as well.). For example, does it account for the time value of money? (Yes!)

Note 1:

If you have some difficulty with the solution for "R," here are the algebraic transpositions, step by step:

$$\$1\ (1 + R)^{5} = \$2$$

$$(1 + R)\ 5 = 2 / 1$$

$$1 + R = (2)^{1/5}$$

$$R = 1.1487 - 1$$

$$R = 0.1487$$

Note 2:

This corresponds with the general TVM formulae:

$$FV = PV\ (1 + R)^{n} (\textit{Basic TVM Formula})$$

AND

$$FV \div (1 + R)^{n} - PV = 0 = NPV (\text{NPV Formula})$$

…and by doing simple algebra (with the first formula above), we get:

$$FV / PV = (1 + R)^{n}$$

$$\mathbf{R = (FV / PV)^{1/n} - 1}$$

Thus, we convert a series of cash flows into what looks like its equivalent Zero-Coupon Bond. This particular value for "R" is referred to as the "Internal Rate of Return," or simply "IRR."

1.21 The Internal Rate of Return for Multiple Cash Flows

Well, that was pretty easy; we had only one cash outflow (PV) and one inflow (FV). What if we had multiple cash inflows? Our equation would look something like the following:

NPV = the discounted values of $\underline{CF_1}$ + $\underline{CF_2}$ + $\underline{CF_3}$ +…. Less initial outlay

= $CF1/(1+R)^{1+}$ $CF2/(1+R)^2$+ $CF3/1+R)^3$ Less initial outlay = NPV

Due to our having here *multiple* unknowns in the denominator, we cannot solve for R using a formula such as above (see Section #1.20), which had only one unknown, i.e., the value of the singular denominator. However, we can guess and guess – until we hit upon the correct answer.

Let's get back to "Example 3." We can agree that we are looking for a discount rate that will cause the NPV to equal zero. In Example 3, we had a positive NPV, so we will have to guess at a *higher* discount rate in order to *lower* the NPV, until it hits zero. So, let's solve for that, by trial and error, *iteratively*, until we get it. We now have "Example #4."

Example #4: – Annuity Cash Flows: One can easily imagine that, for the example (#3) above, there exists a THEORETICAL dollar solution wherein the NPV is *forced* to equal, i.e., where the present value of the project's expected net inflows equals the project's cost. This is true even though the firm's true cost of capital discount rate is different.

In order to discover this other rate, let's call it again the "Internal Rate of Return," or IRR, we must try out several discount rates until we find the correct zero solution; it is an "iterative" or *trial and error* process. There is no direct mathematical solution to this. (You may use the tables….)

Be aware, once again, that in order for the solution to be zero, which is *less* than $1,258.80, we must use a *higher* discount rate than that used above. (Remember the basic rule: rates go up, present values go down.) Let's try 10%, 15%, 20%, 25%. No good? NPV still not zero?

To make your life easy (for the moment), let us try a discount rate of 28%. The rest of the problem is as before (i.e., as in example #3).

Given:

1. Same as Example 3 (except we won't use the cost of capital discount rate!).
2. k = 28% (guessed at – after several failed tries)

Solution:

NPV = (PV $_{\text{annuity factor for 5yrs @ 28\%}}$ for $1,000) – ($2,532)

= [($1,000) (2.532)] – $2,532

= 2,532 – 2,532 = 0

The IRR is 28%!

Conclusion:

If the discount rate (cost of capital) is less than the IRR, the NPV *must* be positive!

1.22 Personal Financial Planning Problem: Net Present Value (In-class Exercise)

Problem: Do you remember Shlomo, the soon-to-be retiree? He was unable to decide based on The Payback Method.

Solution: Shlomo's choices are mutually exclusive. He has to decide between taking benefits now or later. He rightly figures that his expected lifespan is relevant and recognizes that his Payback analysis did not consider that. While it is true that it will take him a somewhat long time to break even, he expects to live a lot longer than that payback period. His parents both lived past 100 and he is in excellent health himself. He expects to live to 100, or 102, at least. He decides to do a (Net) Present Value calculation.

Shlomo assumes a 5% discount rate, based on the Treasury Yield Curve. He, rightly or wrongly, assumes that Social Security payments are as default-safe as Treasury debt. The present value of a 38-year annuity is (PVAF: N = 38; k = 0.05) (12) ($800) = $X, while the Present Value of a 34-year annuity is (PVAF: N = 34; k = 0.05) (12) ($1,200) = $Y, which has to be further discounted by four years to compare it, on a PV basis with his other choice.

Questions:

1. First, calculate these out. You may estimate this by hand, since there is no 34-year row in our tables. Or, you may use a financial calculator.
2. What should he do? Which option should he choose – based on NPV?

Answers:

1. Now: (16.8679) (12) (800) = $161,931. Later: (16.1929) (12) (1,200) = $233,177
2. Wait until later. The NPV is greater in the deferral instance.

1.23 Profitability Index

Another helpful Capital Budgeting technique compares the present value of the project's inflows to its cost or outflows. This is referred to as the "profitability index" ("PI"). It is the ratio of inflows to outflows, calculated on a present value basis. If the PI exceeds one, we may conclude that the expected yield exceeds the discount rate and, accordingly, we would accept any independent project.

$$PI = (PV_{inflows} \div PV_{outflows})$$

Again, if PI > 1.0, we accept the project and necessarily its NPV > 0. Among mutually exclusive projects, we would choose that which has the highest PI. This is a kind of cost-benefit analysis. The PI tells us what we get back relative to each dollar invested, in present value terms.

In cases where there may be negative interim cash flows, we may either add the present value of the negative cash flows to the denominator or subtract them from the numerator.

While the PI is useful in ranking projects, it does not address the problem of scale where competing projects may be concerned. Further, a project may have the highest NPV among its competition, but not the highest PI per dollar invested. The PI does not provide us with a Rate of Return.

The Profitability Index may be used in a "Capital Rationing" context, as seen in the next section.

1.24 The Capital Rationing Problem

There are two reasons why competing projects may be mutually exclusive:

1. There is inadequate space to build two building or house large equipment.
2. There is insufficient financial resources to finance both. We refer to this problem as a "Capital Rationing Problem."

The PI is used to determine which competing projects should be accepted and which rejected. Assume a firm has only finite sources with which to finance some, none, or all potential projects. Those that are accepted provide the highest PI, ranked from highest to lowest.

In the following example, we will assume that the firm has $10,000 of capital to invest.

Project	Investment	NPV	PI
A	3,000	2,000	**1.5**
B	5,000	2,300	**1.4**
C	$2,000	$600	**1.3**
D	2,500	400	1.2
E	4,000	600	1.1

Projects A, B, And C should be accepted since they have the highest PIs, the highest aggregate NPVs, and exhaust the firm's available investment capital.

Suppose instead, we assume that Project D has a higher NPV than originally assumed – equal to $800 with a PI of 1.35.

Project	Investment	NPV	PI
A	3,000	**2,000**	1.5
B	5,000	**2,300**	1.4
C	$2,000	$600	1.3
D	2,500	**800**	1.35
E	4,000	600	1.1

The PI still directs us to select A, B, and C; however, if we assume that the firm's objective is to maximize wealth,

as defined by the projects' NPVs, we should choose A, B, and D. This combination would put the firm over its capital limitation. Clearly, in the presence of limited capital, the PI method is not necessarily consistent with the notion of wealth maximization, which is paramount (to most corporate executives).

In order to maximize the firm's wealth by choosing investment projects with high NPVs, the firm would want to add on more projects if the chosen highest PI project, or projects, do not exhaust the available investment capital. For instance, if the firm has $1,000 to invest and has chosen a project with the highest PI and an investment cost of $900, there is clearly little left to invest. The firm may then choose another project whose cost is, say $100 (i.e., if there is such an opportunity), but has a far lower ranking on the PI range, leaving more desirable projects, in terms of PI rank, "on the table."

1.25 Capital Budgeting and the Limits of a Firm's Capital

With regard to the "The Capital Rationing Problem" we considered the fact that the firm had limited capital resources upon which to call. Here, we revisit the issue given the theoretical possibility that the firm has unlimited (or virtually unlimited) resources and, if so, how that fact would impact, if at all, the Capital Rationing Problem? You will find that it is not merely a theoretical matter.

Question:

If a firm has numerous projects all of which exceed the acceptance criteria under either NPV, IRR, or MIRR (this last concept will come up later), why choose only that which is best under the relevant, or chosen, criterion, assuming mutual exclusivity? (Let's assume there are no physical limitations.) If the firm has abundant access to the public capital markets (even if limited), both bond and stock markets, why not take on and finance all acceptable projects, if need be, with external funds? (In effect, there would be no mutual exclusivity were there no physical or financial limitations!)

Answers:

Is this question realistic? Perhaps so, within reasonable but extensive limits. Too much debt negatively impacts solvency. So.. why not use a combination of debt and equity, so their solvency is not potentially threatened? Firms are reluctant to issue new equity:

- Some investors may see this as a negative "signal" to the market that the firm is not doing well, or that the stock is over-valued. This could cause the stock price to decline.
- More equity could dilute management's ownership stake and control. And EPS may be reduced.
- If they accept projects with low rates of return, i.e., lower than their investors' opportunity costs, investors may not put up any capital!

Internal analysts who recommend investments may have a personal bias. One can get emotionally attached to an idea. One may wish to take a chance on the aspiration that he will look good and thereby enhance his/her career. Forecasts may be too optimistic. Managers know this. By investing in optimal selections, management imposes discipline in the capital budgeting process.

1.26 The Internal Rate of Return (IRR)

Definition of IRR: The IRR is the theoretical discount rate which causes the Initial Outflow to be equal to the PV of Future Net Inflows; in other words, the IRR is the point at which the NPV = 0. This does not mean that the NPV is actually zero. The "true" NPV for a project is based on the cost of capital as the discount rate. (For further definition of IRR, see "What does 'IRR' Mean" below.)

The decision rule for IRR is to choose only those (independent) projects whose IRR exceeds the firm's cost of capital, which was used as the discount rate in calculating the NPV. Choose the mutually exclusive project whose IRR is greatest, provided the IRR exceeds the firm's cost of capital, i.e., the project's "hurdle rate." The hurdle rate is the minimum expected return management will accept in order to invest in a given project, based on the IRR method.

If the project IRR is less than the cost of capital it means that the project will lose money.

If the IRR is greater than the cost of capital (which is the discount rate for the NPV), the NPV will be positive, i.e., greater than zero. Mathematically, if the NPV is positive, the IRR must exceed the cost of capital or discount rate – because the rate must be increased in order to arrive at an NPV of zero.

Illustration of IRR

Note:

Calculating the IRR is accomplished via an iterative, i.e., trial and error process. There is no mathematical solution for IRR (except in the sole instance involving only one future negative cash flow, in which case there is a solvable, quadratic solution).

Question: Given a series of cash flows, how would you know whether to initially choose a large or a small discount rate in order to discover the IRR? The possibilities are virtually infinite! Here is a problem to work on:

Initial Outlay:	$1,000	Cash Flow 1:	$452
Cash Flow 2:	$500	Cash Flow 3:	$278

Answer: Let's first assume an IRR discount rate of zero. The simple sum of the inflows is $1,230. This means that the simple, non-discounted, (internal-) rate of return is (1,230 ÷ 1,000) – 1 = 23%. Indeed, this would hold true if the discount rate were 0%! Here is the formal calculation: $[452 / (1 + 0)^1] + [500 / 1.0^2] + [278 / 1.0^3] \div 1{,}000 - 1 =$ **0.23**.

(Note that this calculation is similar to the *Holding Period Return*.)

The first guess for the IRR must, therefore, be greater than zero in order to decrease the NPV (and hence arrive at the IRR), and also less than 23% – because we shall be discounting each of the numbers, resulting in a smaller

outcome. (The NPV is $230.) Let's try 15% for our first iteration; it's right in the middle of our new range i.e., 0% to 23%). Notice how substantially we have narrowed the range of our guesses! (With a discount rate of zero, the Rate of Return = 23% and the PI = 1.23.)

1st Iteration: k = 15%

Year	Cash Flow	PVF	PVCF
0	*($1,000)*		
1	*452*		
2	*500*		
3	*278*		
		NPV=	

Since this clearly does not provide us with the answer we are looking for, we need to try again. (We are looking for the single discount rate, which will yield: NPV = 0. Should we raise or lower the discount rate from 0.23? Let's see.

Since the first guess was no good, here is the next try – or iteration.

We must lower the discount rate in order to raise the NPV (from the negative solution at which we arrived).

2nd Iteration: k = 12%

Year	Cash Flow	PVF	PVCF
0	*($1,000)*		
1	*452*		
2	*500*		
3	*278*		
		NPV=	

Some Difficulties with the IRR

Project Scale: We could have two competing projects, each with radically different costs, and potentially choose one based on the IRR solution alone – without giving any heed to the very different costs involved. From a business perspective, the choice may not be optimal.

Negative Future Cash Flows: In some projects, it is possible that we could get an interim, future cash flow that is negative. This may be due to having to engage in a major and expensive equipment overhaul at a specific point during the project's life. Cases wherein there may be "negative interim FCFs" may yield a "quadratic" solution. As you may recall from high school mathematics, such cases make possible two equally valid solutions. This presents a problem for the decision maker, i.e., which solution, if either, should be used in making the decision? See the pages below for analysis.

Reinvestment Rate Assumption: As we will soon see, the IRR implies that the pro-forma cash flows are –

themselves – being reinvested as received at the calculated IRR; this is a kind of mathematical redundancy, which, for numerous reasons, cannot be valid. See below for our in-depth analysis.

The following are the solutions to the two iterations attempted above.

1st iteration: 15%.

Year	Cash Flow	PVF	PVCF
0	*($1,000)*	1.0000	($1,000)
1	*452*	.8696	393.06
2	*500*	.7561	378.05
3	*278*	.6575	182.79
		NPV=	($46.11)

2nd iteration: 12%.

Year	Cash Flow	PVF	PVCF
0	*($1,000)*	1.0000	($1,000)
1	*452*	.8929	403.59
2	*500*	.7972	398.60
3	*278*	.7118	197.88
		NPV=	—

Notes:

- When IRR > k, we accept the project – by mathematical rule.
- If IRR > k, the NPV calculated at k must be positive; the project is projected to be profitable, i.e., to add to firm wealth.
- If the NPV *exceeds* zero, then the IRR discount rate must be *increased* in order to *make* the NPV = 0. And vice versa. That is one of our basic TVM rules.
- Thus, for any given project, the IRR and NPVs rules will be consistent with one another.

1.27 Some more NPV and IRR Practice Problems

Problem #1:

Cost = $61,446

Cash Flow = $10,000 per year

N = 10 Years

NPV @ 5% discount rate = ?

Possible Range for IRR:

IRR = ?

Problem #2:

Cost = $1.5 million

Project Life = 5 Years

k = 3%

CF_1 = $725,000

CF_2 = $830,000

CF_3 = $840,000

CF_4 = $625,000

CF_5 = $225,000

Salvage Value = $35,000

Calculate the NPV!

1.28 IRR Practice Problems (Solutions)

Problem #1:

NPV= $10,000 x (PVAF 5%; n = 10) – Cost

= ($10,000) (7.7217) – $61,446

= 77,217 – 61,446

= $**15,771**

As we did before, we will start out assuming a 0% discount rate. Thus, we simply add the ten $10,000 payments and divide by the cost of $61,446 (less one): [$100,000 ÷ $61,446] – 1 = 0.627.

Possible Range for IRR: 5% to 62.7%

Since the NPV at a 5% discount rate was positive, the IRR must be greater than that. The upper limit is 62.7%. This is a wide range. You could start guessing, using perhaps a first guess of 30% and taking it from there. Alternatively, if you have tables, you might try using them!

Since we are dealing with a round number of $10,000, you could look across the Present Value Annuity table – on the 10-period row – for a multiplier that comes closest to being a multiple of the cost figure ($61,446). A multiple of exactly 6.1446 appears in the 10% column! Thus, ($10,000) (6.1446) = $61,446, making the NPV = 0.

Mathematically the solution for this is: ($10,000) (PVAF) = $61,446; PVAF = 6.1446.

IRR= 10%

Problem #2:

Period	Free Cash Flow	PVF @ 3%	PVCF
0	($1,500,000)	1.0000	($1,500,000)
1	725,000	0.9709	703,902.5
2	830,000	0.9426	782,358.0
3	840,000	0.9251	768,684.0
4	625,000	0.8885	555,312.5
5	225,000 + 35,000	0.8626	224,276.0
		NPV=	$1,534,533

1.29 What Does "IRR" Mean? (A Brief Review)

In our earlier definition of IRR, we said that the IRR is the discount rate, which causes the NPV to be equal to zero. For present purposes, think of IRR in its mathematically equivalent expression, that is once again, as the discount rate, which causes the present value of the future cash flows to equal the project's initial cost, or investment. Thus, you will find that IRR is a return on investment (where investment = cost) calculation. Think of this in the following manner.

If you invested $1 in cash, and in five years it grew to $2, that would imply a certain rate of return, or return on investment (where the investment is $1), based on a calculation involving the time value of money. Mathematically, this statement may be written as $1 $(1+R)^5$ = $2. You can then solve for R and will find that it is 0.1487 (approximately 15%.) This math is based on TVM, as below:

$$\mathbf{PV\ (1 + R)^{n} = FV}$$

Let's say that R = IRR

And therefore:

$$\mathbf{FV / PV = (1 + IRR)^{n}}$$

$$\mathbf{(FV / PV)^{1/n} = 1 + IRR}$$

$$\mathbf{(FV / PV)^{1/n} - 1 = IRR}$$

"R" is the annual compounded rate of return. Nothing we didn't already know here. In our example, it means that your return on investment is about 15%, that the $1 grows to $2 in five years at a rate of 15%, and, finally, that the present value of $2 discounted at 15% is equal to $1. Thus at 15%, the investment cost ($1) and the present value of the future, in this case, singular, cash flow ($2) are the same; this is the same as saying that the NPV equals zero, a condition which holds when cost/investment less PV of future cash flows equals zero. This does not say that you earned nothing. You earned 15%!

This is as true for a series of cash flows, whether even or uneven, as it is for one initial outflow (say $1, as in the former "$1 to $2" example) and one "terminal inflow" (say $2). Actually, by calculating the future, or terminal-value of the cash inflows, we are "translating" the multiple cash flows into its equivalent single dollar value, such as $1.

Thus, IRR is the discount rate, which equates the present value of an investment's (multiple) future cash flows with its initial cost (or investment). So, "IRR" is a kind of rate of return, or "return on investment" measure, utilizing the time value of money. Later we will discuss why it is viewed as an "internal" rate of return measure, what the phrase "internal" means, and what problems may be associated with the IRR measure.

Soon, we will see if we can calculate the IRR for multiple cash inflows, while importing an “external” reinvestment rate, which may – or may not – be different than the originally calculated IRR. First, we shall have some more ground to cover relative to certain IRR nuances.

1.30 The NPV vs. the IRR: Differences in Methodologies (Summary and Review)

NPV:

- Use of the firm's "cost of capital" as the discount rate.
    - We may think of the cost of capital as an external rate, i.e., external to the formula.
    - For now, the cost of capital has been "given."
- Accept only those projects whose NPV > $0; reject all others.
- If given two or more competing, or "mutually exclusive," projects, choose that project, which provides the highest NPV (assuming its NPV > $0).

IRR:

- Calculate the IRR through an "iterative," Trial and Error process.
    - If the initial NPV > 0, you must raise the discount rate.
    - The discount rate, where NPV = 0, is the IRR
- Choose only those independent projects whose IRR exceeds the stated cost of capital;
    - This also means that NPV > 0
- If given mutually exclusive projects, choose that project which has the highest IRR (provided the IRR exceeds the firm's cost of capital)

Note:

Let's say that at a 10% cost of capital / discount rate, a project's NPV is $1,000. In order to get the NPV "down" to zero, the rate must be raised. Thus, if a project's IRR exceeds the firm's cost of capital, the project's NPV, using the cost of capital as the discount rate – mathematically – must be positive.

1.31 Project Scale

Below we see two mutually exclusive projects, one of which requires greater investment and thus also, as it were, produces annual higher FCFs, – and NPV. The other has a smaller investment, but higher FCFs *relative* to the outlay – and a higher IRR. Put differently, the outlay for "A" is 50% higher than for "B," the smaller project. However, the FCFs for "A" are only about 43% greater than for "B."

We will use 10% as the discount rate for the NPV calculation. Note the conflict between the two indicated rules of thumb as below.

Year	**FCF Project A**	**FCF Project B**
0	($3,000)	($2,000)
1	1,000	700
2	1,000	700
3	1,000	700
4	1,000	700
5	1,000	700

NPV	$790.80	$653.55
IRR	19.86%	22.11%
PI	1.26	1.32

IRR and PI say accept "B," while NPV says accept "A."

The IRR is a measure of return, while the NPV provides a measure of the expected dollar increase in wealth. The NPV is largely affected by scale. The IRR and PI will yield consistent results with one another; these methods are more affected by inflows relative to outflows.

What should the company do?

1.32 Multiple IRRs

As briefly noted earlier, when one (or more) of the *interim* cash flows is negative, there may be more than one mathematically correct IRR. If in such cases, there are two future cash flows, one of which is negative, we will get a "quadratic" solution. By definition, a quadratic equation is one that contains three exponents: zero, one, and two.

	Cash Flows
0	($1,600)
1	$10,000
2	($10,000)

As we know, the IRR is the rate at which the NPV will be equal to zero. Therefore:

$NPV = 0 = [(\$1{,}600) \div (1 + IRR)^0] + [\$10{,}000 \div (1 + IRR)^1] + [(\$10{,}000) \div (1 + IRR)^2]$

Where IRR = 25% and 400%!

If we just used the NPV approach, we would not have a problem because the discount rate would be given.

Solutions/Proofs:

25%: $0 = [(1{,}600) \div 1] + [10{,}000 \div 1.25] + [(10{,}000) \div 1.25^2]$

$0 = (1{,}600) + 8{,}000 + (6{,}400) = 0$

400% $0 = [(1{,}600) \div 1] + [10{,}000 \div 5] + [(10{,}000) \div 25]$

$0 = (1{,}600) + 2{,}000 + (400) = 0$

What if the capital costs were just 10% ($k = 0.10$)? Wouldn't we accept this project since the costs are less than each quadratic solution? No! If you take the time to calculate the NPV, you will find that it is negative, i.e., ($773)! If you also calculate the NPV using other discount rates, e.g., 20%, 30%, 40%, and 50%, you will find that the NPV goes back and forth from positive to negative. In short, we cannot use the IRR at all when an interim future cash flow is negative! In other words, we reject the … IRR method!

1.33 Quadratic Solution to IRR

As just observed, when a project reflects (multiple) negative interim future cash flows, we get a "quadratic solution," yielding multiple IRRs. Suppose a firm, for example, has a cost of capital, or "hurdle rate," of 15%, but the dual quadratic solutions reflect both a 10% and a 20% IRR. Do you accept or reject the project? Most probably, you should utilize an alternative decision rule. The following example illustrates this troubling outcome.

Year	**0**	**1**	**2**
FCF	($1,000)	$3,200	($2,400)

This is equivalent to:

$$[\$3{,}200 / (1 + IRR)^1] – [\$2{,}400 / (1 + IRR)^2] – \$1{,}000 = 0$$

A quadratic formula will have three constants and two variables. We will convert the formula created above to its quadratic form as in:

$$ax^2 + bx^1 + c = 0$$

First, to simplify, let's decimalize (just for this moment) the numerator.

$$[\$3.2 / (1 + IRR)^1] – [\$2.4 / (1 + IRR)^2] – \$1 = 0$$

Next, we will first substitute "x" for "1 + IRR."

$$[\$3.2 / (x)^1] – [\$2.4 / (x)^2] – \$1 = 0$$

In multiplying the expressions by x^2, we rid ourselves of the divisor:

$$3.2\ (x)^1 – 2.4 – 1\ (x)^2 = 0$$

$$3.2x – 2.4 – x^2 = 0 =$$

Last, we reorder *and* change ALL the signs (this is equivalent to moving the expression to the other side of the equal sign) so that our formula now looks like a quadratic:

$$x^2 – 3.2x + 2.4 = 0$$

Now, we can solve for x.

This is your standard quadratic equation the solution for which (i.e., solving for "x"):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3.2 \pm \sqrt{3.2^2 - 4(1)(2.4)}}{2(1)}$$

$$= \frac{3.2 \pm 0.8}{2}$$

$$= 2.0 \ \ or \ \ 1.2$$

This “famous” general quadratic solution is simply the formula, which solves for “x.”

Finally, since we substituted “x” for “1 + IRR,” the IRR = x – 1. Thus, we get two possible IRRs: 100% *AND* 20%! Which is “correct”? Both? Mathematically, yes! For us in this context? It i snot helpful! We must therefore reject the method.

It should be noted that the quadratic solution works only if there are just two future cash flows of which one is negative; if, as is likely, there are more than two, the solution cannot conform to the quadratic formula (i.e., $x^2 + x^1 + 1 = 0$) and its famous solution.

As it works out in Mathematics, every time we get a negative interim cash flow, we get an additional IRR. In this example, we had two negative numbers – ($1,000) and ($2,400), hence we observed two IRRs. Had there been only one negative cash flow, say the initial outlay, we would have had only one IRR! It is not terribly unusual for a project to have a negative future cash flow. Often, as an example, the equipment in which the company has invested will require a major overhaul at some point, causing that year’s FCF to come in negative.

As an exercise, calculate the project’s NPV using 15% and, alternatively, 25% as discount rates. Under this alternative rule of thumb, would you accept or reject the project?

1.34 Quadratic Practice Problems

Practice Problems:

Students will be asked in class to solve the following problems. Work in teams. Some space is provided here to work. (Solutions are not provided.)

Problem #1

Year	0	1	2
FCF	($500)	33,200	($1,400)

$(- 500) / (1 + IRR)^{0} + (2,200 / (1 + IRR)^{1} – (1,400 / (1 + IRR)^{2} = 500\ x^2 – 2,200\ x + 1,400$

Here, you must employ the Quadratic formula.

$X_1 = 0.7717$ $IRR_1 = 0.7717 – 1 = (0.23)$

$X_2 = 3.6283$ $IRR_2 = 3.6283 – 1 = 2.62$, i.e., 262%

Problem #2

Year	0	1	2
FCF	($1,000)	$6,300	($4,400)

$(- 1,000) / (1 + IRR)^{0} + (6,300 / (1 + IRR)^{1} + (- 4,400 / (1 + IRR)^{2} = 1,000\ x^2 – 6,300\ x + 4,400$

$X_1 = 5.5$ $IRR_1 = 5.5 – 1 = 4.5$, i.e., 450%

$X_2 = 0.8$ $IRR_2 = 0.8 – 1 = (0.20)$

Problem #3

Year	0	1	2
FCF	($800)	$4,600	($3,650)

$(- 800) / (1 + IRR)^{0} + (4{,}600 / (1 + IRR)^{1} + (- 3{,}650 / (1 + IRR)^{2} = 800\ x^{2} – 4{,}600\ x + (-3{,}650)$

$X_1 = 4.80$ $IRR_1 = 4.80 – 1 = 3.8$, i.e., 380%

$X_2 = 0.95$ $IRR_2 = 0.95 – 1 = (0.05)$

1.35 The Reinvestment Rate Assumption of the Internal Rate of Return

Problem:

The IRR calculation assumes that the future cash flows will each be reinvested, to the horizon, at the project's calculated IRR rate. This reinvestment rate assumption is most likely unrealistic because, in fact, the firm may choose to invest interim cash flows in some other project or investment. Should this be true, the project's true rate of return will be more or less than the IRR, depending on whether the reinvestment rate is greater or less than the IRR.

To understand the nature of this problem, let's recalculate the IRR formula using the IRR as the reinvestment rate, which will thereby enable us, first, to figure the project's future – or "terminal" – value. (If you wish, it may be helpful to draw a timeline.) Then, we will compare the terminal value to the project's initial cost (or investment value), which using TVM, will imply a discounted rate of return. If this discounted rate of return is the same as the IRR, it will mean that the IRR calculation embeds the redundancy implicit in the assumption that the IRR reinvests the interim cash flows at the IRR.

Let's show the mathematics of this problem (so as to be sure that it is correct as stated), using **"Example 3"** above:

If the project's reinvestment rate = 28% (i.e., the IRR), we have the following:

Project's Future Value (FV)	= CF × FVAF
	= $1,000 × 8.6999
Project's Cost	= $2,532
Implied Rate of Return	= $8,699 ÷ $2,532
	= 3.44×

This means that for every dollar ($1) invested, the project will produce, at the horizon or terminal point, $3.44 – assuming a reinvestment rate equal to the IRR, which in this instance is 28%. We can solve for the annual compounded rate or return as follows:

$$(3.44)^{1/5} - 1 = .28$$

Alternatively, using the interest rate tables, one would look for the 3.44 multiplier along the five-period row in the simple FV chart; this cell will be (approximately) located under the 28% column.

In any event, this implies a rate of return of 28%, i.e., the same as the originally derived IRR! In other words, the problem with the IRR is that it assumes reinvestment at the IRR, an unlikely possibility!!

The general IRR formula is now $\mathbf{R = [(FV / PV)]^{1/(n \times p)} - 1}$.

1.36 The Reinvestment Rate and the MIRR (The Idea)

What if the reinvestment rate assumption were not the same as the IRR, e.g., 10%, which is *less* than the IRR? Then:

FV = $1,000 × 6.105

PV (cost) = $2,532

FV ÷ PV = $6,105 ÷ $2,532 = 2.411×

For every dollar invested, I will have at the horizon, $2.411, *and* using the formula:

$$(FV / PV)^{1/(n \times p)} - 1 =$$

$$[(2.411)^{1/5} - 1] = 0.1924.$$

This implies a rate of return of about 19.24%, i.e., lower than the IRR. This new, 19.24% return is called the Modified IRR, or MIRR. This is a more accurate measure of return, given the input of a more realistic reinvestment assumption, than our having arbitrarily assumed a reinvestment rate for the future cash flows at the already calculated IRR.

Another way of looking at this (not mentioned above), is by equating the present value of the future inflows with the project's cost, i.e., by setting the NPV to equal zero:

$$\$6,105 \div (1.1924)^5 = \$2,532$$

In summary, when the reinvestment rate ("REIN") is the same as the IRR, the IRR and the MIRR will be the same. In most instances, the reinvestment rate will be less than the IRR, because the firm will have less attractive opportunities, as compared to the project at hand, which was accepted due to its high expected return; this need not always be true however. The important relationships are summarized below:

if REIN = IRR *then* IRR = MIRR
if REIN < IRR *then* MIRR < IRR (and vice versa)

MIRR Rule:

As with the IRR, accept all independent projects whose MIRR > k (where k is the hurdle rate, cost of capital); amongst competing, mutually exclusive projects, accept that which has the highest MIRR. Neither the IRR nor the MIRR addresses the issue of project size.

Notes:

- To make matters worse, the reinvestment rate does not have to be the same each year; it may be a variable. We have not presented that possibility here.
- The IRR will be the same as the MIRR when there are no interim cash flows, thus requiring no reinvestment. This would be the corporate project equivalent to a zero-coupon bond, i.e., one outflow and inflow each.
- One reason the MIRR method is so attractive is that it does not have the "multiple solutions" issue that the IRR has, as will be illustrated below.

1.37 General MIRR Formula (Derivation)

The general formula for MIRR conforms to our basic TVM formula:

$$PV\ (1+R) = FV \quad \text{or} \quad [\ (FV \div PV)^{1/n}] - 1 = R$$

While this works well for a case involving just one outflow and one inflow, we will usually have multiple inflows. Thus, a more general formula, which you may use for the MIRR is:

$$(\text{Future Value} \div \text{Present Value of Cost})^{1/n} - 1 = \text{MIRR}$$

Where the Future Value ("FV") is:

$$\Sigma\ [CFs\ (1 + REIN)^{n}] = FV$$

The project's Terminal Value ("TV") is the future value ("FV") of the reinvested cash flows the project is expected to produce, assuming an "exogenous" (i.e., external) reinvestment rate, to the horizon.

Where "REIN" is the compound reinvestment rate for the cash inflows, and the "terminal value" is thereby determined by compounding the interim cash flows at the reinvestment rate to the horizon.

Again, the CFs are the nominal future cash flows, each of which is bumped up respectively to its terminal value and then aggregated. In calculating the terminal values, be careful about the manner in which you are employing exponents when dealing especially with uneven cash flows; if the cash inflows are an annuity, the process is much simpler. Remember you must figure how many periods are left to the horizon for each cash flow; "the arrows are going to the right."

The "present value of the cost" (PV) is the original cost of the investment, or its initial outlay.

Or more simply: $[(FV \div PV)^{1/n}] - 1 = \text{MIRR}$

We know this formula from the "$1 to $2" discussion. It is the same formula as used there!

Problem: Suppose someone had $3,050 nine years ago and today it has grown to $3,950. What is his compound annual rate of return?

Solution: $(3{,}950 \div 3{,}050)^{1/9} - 1 = 0.0291$

Note that, in this problem, the IRR = MIRR because there is no cash flow to reinvest.

1.38 The Modified Internal Rate of Return (MIRR) (Problem)

Illustration of MIRR: Uneven Cash Flows

Given:

1. Initial Outlay = $1,000

2. Net nominal expected cash inflows as below

3. Reinvestment Rate = .10

Question: What is the MIRR? [*Warning*: Be sure you get the exponents correct!]

Year	Nominal Cash Inflows	FVF @ 10%	Future Values
1	$500		
2	$400		
3	$300		
4	$100		
		Terminal Value=	

Note:

The correct directionality of the respective future value factors and the correct number of periods in completing the FVF column. Remember that you are "bumping up" the nominal cash flows to the horizon. A timeline may be useful. Below is a five-year timeline; do your own for this (four-year) problem.


*
0
1
2
3
4
5

1.39 MIRR Solution

Period	Nominal Net Cash Inflow	FVF @ 10%	Future Values
1	$500	1.331	$665.50
2	$400	1.210	484.00
3	$300	1.100	330.00
4	$100	1.000	100.00
		Terminal Value=	**$1,579.50**

$(1{,}579 \div 1000)^{1/4} - 1 = 12.11\%$

Question: What if the cash flows occurred at the start (rather than the end) of each period?

Solution: In reality, we spread projections using end-of-period assumption – even though the flows are received over the course of the period and not discretely, in one fell swoop at the end alone. Still this exercise has some interesting numerical implications, as you shall see. In order to solve this, we may employ the same method used in converting an ordinary annuity to an annuity due. Specifically,

$$\{[(1{,}579.50)\ (1.10)^1] \div 1{,}000\}^{1/4} \cong 15\%$$

This "short-cut approach" may seem counter-intuitive, or just incorrect, because this example is clearly not an annuity. To prove out the validity of the short-cut used, let's do it the long way. If we get the same answer, the short-cut was correct.

Period	Nominal Net Cash Inflow	FVF @ 10%	Future Values
1	$500	1.4641	$732.05
2	$400	1.331	532.40
3	$300	1.2100	363.00
4	$100	1.1000	110.00
		Terminal Value =	**$1,737.45**

We get the same terminal value, multiplier, and return!

Note the MIRR is a.k.a. *External* Rate of Return (ERR)!

1.40 Problems with Capital Budgeting Methods (Summary)

TVM: This budgeting method includes the time value of money.

Later CFs: Cash flows after payback are considered.

PS: The project's relative scale of its cash flows are evaluated

ROR: The method provides for a rate of return calculation.

Reinv. Assumption: This method does – or does not – embody a "redundancy" problem.

Capital Budgeting Method	Rule of Thumb	Accounts for TVM	Accounts for Later Cash Flows	Provides for ROR Measure	Accounts for Project Scale (PS)	Reinvest. Assumption
Payback Method						
Discounted Payback Method						
Net Present Value (NPV)						
Profitability Index						
Internal Rate of Return						
Modified Internal Rate of Return (MIRR)						

Note:

The foregoing analysis assumes all cash flows are certain. What about *risk*? Can anyone be certain that the projections are correct?

1.41 Review Questions: Chapter One

Review Questions: Chapter One

1. You are given the following data. Calculate the Payback.

Initial Cost= $12,500

CF_1= $7,500	CF_5= $4,750
CF_2= $6,275	CF_6= $4,650
CF_3= $4,000	CF_7= $ 2,150
CF_4= $4,850	CF_8=$ 0

2. What problems can you cite with respect to the use of the Payback method?

3. Assuming a 2% discount rate, what is the Discounted Payback – from the prior question?

4. What problems can you cite with respect to the use of the Discounted Payback method?

5. Assuming a 3% discount rate, what is the Net Present Value?

6. What problems can you cite with respect to the use of the Net Present Value method?

7. Assuming a 4% Discount Rate, what is the Profitability Index, given the following?

Initial Cost = $22,545

CF_1= $7,500	CF_5= $4,750
CF_2= $6,275	CF_6=$4,650
CF_3= $4,000	CF_7=$2,150
CF_4=$4,850	CF_8=$0

8. Assuming a 4% Reinvestment Rate, what is the project's Modified Internal Rate of Return?

9. What problem(s) pertain with regard to two projects' disparate relative Scales?

10. You are given the following data. Calculate the project's Internal Rate of Return.

Initial Cost= $13,621.80 $CF_{1\text{-}15}$ = $2,000

11. Try to make up an IRR problem using Uneven Cash Flows.

12. State the rules of thumb for all the above Capital Budgeting Decision-making methods.

13. We have not, in this review, dealt with the risk of the Future Cash Flow projections. They most likely will not come in as projected. How do you propose that that issue be dealt with?

14. Do the projected numbers tell the whole story? Why or why not?

15. Can you make up another chapter question not included above?

1.42 Selected Solutions to Chapter One Problems

Question #1

The Payback is just over 3 years, as per the table below.

	FCF	Cum. FCF
0	(12,500)	(12,500)
1	3,500	(9,000)
2	3,250	(5,750)
3	3,000	(2,750)
4	2,800	50
5	2,750	
6	2,650	
7	2,000	
8	0	

Question #3

See if you can fill it in below.

	FCF	PVF	PVCF	Cum. PVCF
0	(12,500)			
1	3,500			
2	3,250			
3	3,000			
4	2,800			
5	2,750			
6	2,650			
7	2,000			
8	0			

Chapter 2: Advanced Capital Budgeting

2.1 Chapter Two Learning Outcomes

Learning Objectives:

- **Compare** IRR versus NPV for projects with radically different cash flow series.
- **Explore** possible solutions when negative interim returns are involved.
- **Consider** an appropriate method that addresses the issue of Unequal Lives for competing projects.
- **Recognize** the potential effect of a firm's Capital Limitations on its investment choices.
- **Calculate** Spot Rates and possibly utilize them in Capital Budgeting.
- **Debat**e the usefulness of the *Market Value Added* measure for management performance analysis.

2.2 Comparison of NPV and IRR: Some Technical Points

In certain circumstances, the analyst will find that the choice amongst two competing projects will depend on how high or low the discount rate is when utilizing the NPV and IRR methods. For instance, you are given the *free cash flows* for the two projects, A and B, as below. For each, resolve the questions below.

	Project A	**Project B**
0	($300)	($405)
1	(387)	134
2	(193)	134
3	(100)	134
4	600	134
5	600	134
6	850	134
7	(180)	0

Rates	**NPV Profiles for**	
	Project A	**Project B**
0.0		
10.0		
12.0		
18.1		
20.0		
24.0		
30.0		

This NPV Profiles worksheet provides room for the final NPV number only; it does not allow space for the calculations needed to arrive at the NPVs for each different discount rate. You may, therefore, wish to create a spreadsheet similar to that, which was used for "Uneven Cash Flows" in order to complete this exercise.

Questions Coming up:

- What are the IRRs for each project?
- What is the "crossover rate" for projects A and B?
    - The crossover rate is the discount rate at which the NPVs of each project will be the same.
- What is the significance of the "crossover rate"?
- Construct a diagram of the two projects' respective NPVs as they vary across discount rates. This will illustrate visually the crossover rate. (*See NPV Profiles Illustrated below*.)

- If the reinvestment rate is 12%, what are the MIRRs for each project?

2.3 Solution to Problem NPV vs. IRR

We can calculate the NPV Profiles for each project as found below. The NPV Profile depicts how the NPV for a project will change when assuming a set of different discount rates.

	NPV Profiles for	
	NPV_A	**NPV_B**
0.0	$890	$399
10.0	283	179
12.0	200	146
18.1	**000**	62
20.0	(49)	42
24.0	(138)	**00**
30.0	(238)	(51)

The "NPV Profiles" are the sets of NPVs at varying discount rates.

First, take note that the IRRs for the two projects are as follows:

$$IRR_A = \textbf{.181}$$

$$IRR_B = \textbf{.240}$$

We recognize that, in truth, there will be multiple IRRs – due to the presence of negative free cash flow projections. For present purposes, we will use only those IRRs that "fit" in the range denoted in the table.

You may also have observed that the "crossover rate" will be between 12% – 18.1% (note that the IRR numbers have been marked above in bold). The crossover rate is the discount rate at which the preference for one project over another, on an NPV basis, changes from one project to the other – as you go from lower to higher rates.

This rate is both interesting and relevant because, as may be readily seen, at lower discount rates, project "A" will produce relatively higher NPVs than project "B", while at higher discount rates project "B" will produce higher NPVs. Since the discount rate can change over time, the moment at which one calculates the NPV, may yield an outcome that may be arbitrary. The exact crossover rate is therefore noteworthy. (At this point, you should diagram this table. A few pages hence, you will find a diagram to fill in.)

Let's get back to the crossover rate. To discover the crossover rate, we wish to find that one rate at which the

NPVs for both projects are the same. That is to say mechanically, we will calculate the NPV of the differences between the two projects' respective cash flows to accomplish this mathematical task. Thus, we create a third, fictitious project data set – let's call it "Project C," whose cash flows are as follows:

	Project A	**Project B**	**"Project C" $CF_A - CF_B$**
0	($300)	($405)	105
1	(387)	134	(521)
2	(193)	134	(327)
3	(100)	134	(234)
4	600	134	466
5	600	134	466
6	850	134	716
7	(180)	0	(180)

In other words, we calculate the differences between the cash flows of Projects "A" and "B," and treat those differences as though it were another, third project, which we are calling "Project C" or "$CF_A - CF_B$." We must work iteratively in order to arrive at the correct crossover rate. (We will use just the one rate that falls within a useful range for this illustration, i.e., between 12-18%. We are really not concerned with the problem of multiple IRRs because we are more focused on the NPV.…)

If Project C is the difference in the cash flows of Projects "A" and "B," then the IRR for "C" must produce a difference in the NPVs for the two projects equal to Zero. This is the very crux of the concept being discussed here.

The precise crossover rate for both projects A and B = **.1453**. (And $IRR_C = 0.1453$.) Once again, while there will, of course, be numerous solutions for the IRR, given that there are some negative cash flows; the one solution (of many) that falls within our 12-18.1% percent range is 14.53%.

The crossover rate is the discount rate where the NPV for both projects will be the same, i.e., $NPV_A = NPV_B$. The table below proves that out – with an immaterial rounding error.

$	CF Project A	Discount Rate	PV of CF_A	CF Project B	PV of CF_B
0	(300)	$\div 1.1453^0$	(300)	(405)	(405)
1	(387)	$\div 1.1453^1$	(337.90)	134	117.00
2	(193)	$\div 1.1453^2$	(147.14)	134	102.16
3	(100)	$\div 1.1453^3$	(66.56)	134	89.20
4	600	$\div 1.1453^4$	348.72	134	77.88
5	600	$\div 1.1453^5$	304.48	134	68.00
6	850	$\div 1.1453^6$	376.62	134	59.37
7	(180)	$\div 1.1453^7$	(69.64)	0	0
		NPV=108.58		**NPV=108.61**	

(The numbers in the NPV table were rounded to the nearest cent, so in the end, there is a rounding error of about $0.03; this should not be of any conceptual concern.)

This also says that, at the 0.1453 discount rate, the difference in the two respective projects' NPVs will be zero, i.e., $NPV_A - NPV_B = 0$. Project C represents the differences in the projects' cash flows and thus must also reflect the difference in the two NPVs; in other words, the NPV for Project C should be zero, i.e., $NPV_C = 0$. The rate that causes the NPV of Project C to be zero is its IRR: 0.1453. The following table bears this truth out.

	"Project C" $CF_A - CF_B$	**$\div 1.1453^N$**	**=**
0	105	$\div 1.1453^0$	105.00
1	(521)	$\div 1.1453^1$	(455.78)
2	(327)	$\div 1.1453^2$	(249.29)
3	(234)	$\div 1.1453^3$	(155.76)
4	466	$\div 1.1453^4$	270.84
5	466	$\div 1.1453^5$	236.48
6	716	$\div 1.1453^6$	317.25
7	(180)	$\div 1.1453^7$	(65.53)
		Sum/NPV=	0

To repeat, the "crossover point" depicts the point at which the NPVs for both projects will be the same; that also says that the difference between the two projects' NPVs will be zero. Because "Project C" itself represents the difference in the two projects, the NPV of Project C will be zero, and the IRR of Project C will be the rate that caused the difference in the NPVs to be zero. That rate was calculated to be 0.1453.

So, while the IRR for Project C is 0.1453 and its NPV = 0, it does not mean that the respective NPVs for either projects "A" or "B" will be equal to zero at the IRR for project "C"; the individual NPVs will be positive, as you can see in the graph (below). However, at the IRR for project "C," the NPVs for both projects "A" and "B" will

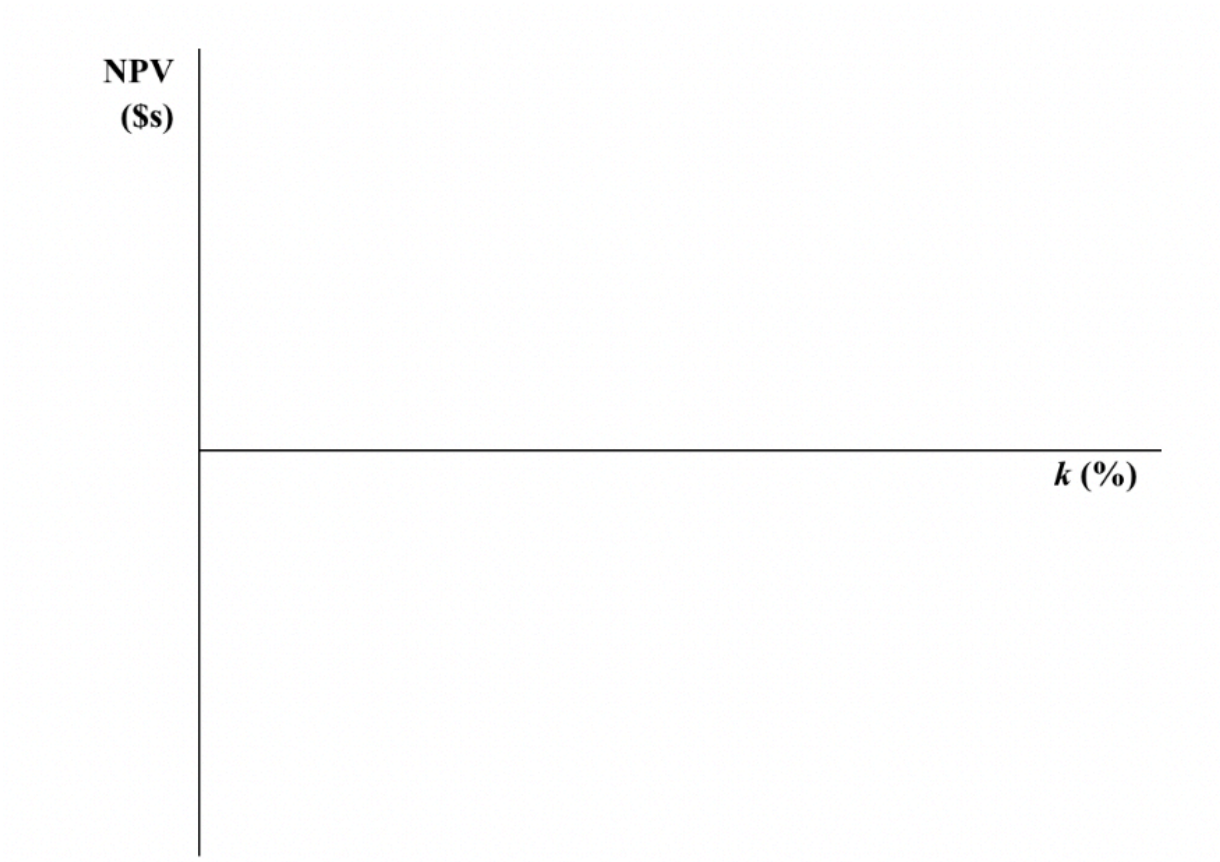
be the same. That is the IRR_C will be the point at which $NPV_A - NPV_B = 0$. Remember: Project "C" represents the difference between the two projects' cash flows.

As a practical matter, an analyst who studies two (or more) competing projects whose cash flow profiles are radically dissimilar, should also calculate the crossover point. Why? If it so happens that the company's cost of capital is very close to the crossover point, say within a percentage point, he may reconsider whether to recommend one project over the other solely based on the decision rule of thumb of choosing the one with the higher NPV. In the case just evaluated, suppose the firm's k = 14% or 15%. Based on this, the analyst, using the rule of thumb, will "flip" his decision from one to the other project. Isn't that just arbitrary? Instead, he should rely on some other rationale, maybe even a qualitative one, heaven forbid!

Since a picture is worth a thousand words, a diagram is presented below. See if you can fill it in (below).

2.4 NPV Profiles Illustrated

On this page, diagram the NPV Profiles for project “A” and “B” at the various discount rates provided above. You will observe that which was discussed above. Take note of the curvature of the lines. The IRR for “Project C” is also the point at which the $NPV_A = NPV_B$.



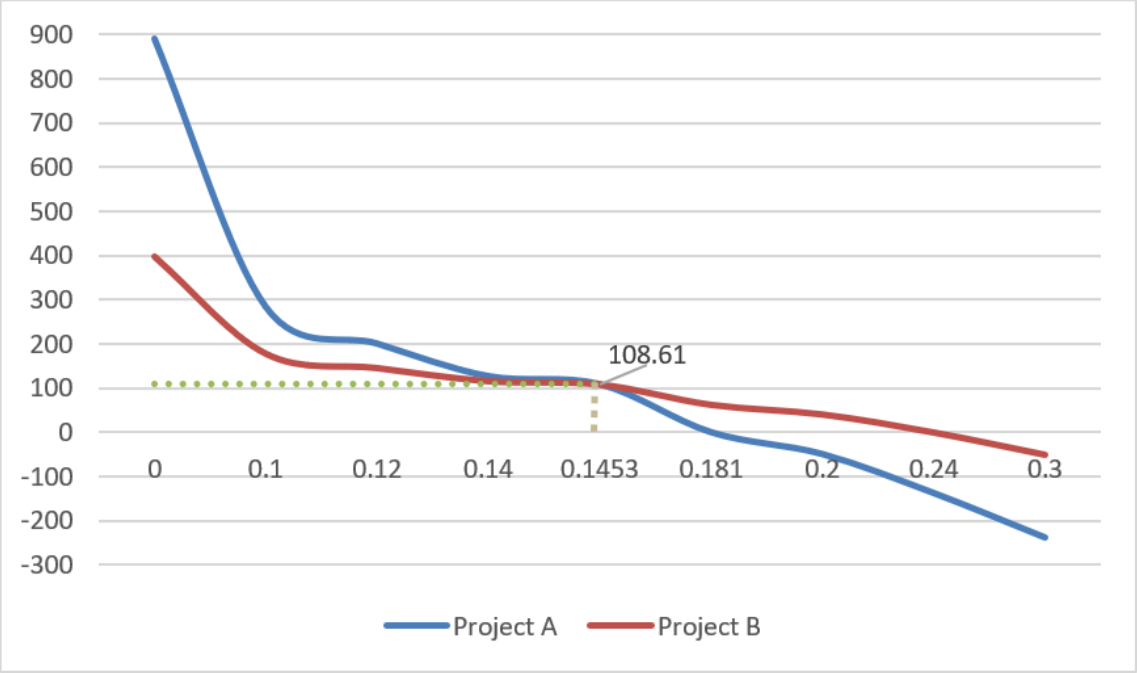
	Accept	Reason
NPV	A	If k < x
	B	If k> x

Here, “k” is the discount rate / cost-of-capital, and “x” is the crossover point.

If the projects are mutually exclusive, and the cost of capital exceeds the crossover rate, both NPV and IRR

methods will yield the same choice, which is to accept "B" and reject "A" – except when k > 24%, in which case the IRR indicates the rejection of both projects. The completed table is found below (dollars on the vertical; rates on the horizontal).

Comparison of Projects A and B[1]



At all discount rates greater than 14.53%, the NPV for "B" shall be higher. At an NPV set to zero, "B" has a higher IRR, i.e., 24% versus 18.1%. If the cost of capital is less than the crossover rate, the two methods will yield conflicting decisions. For example, if the discount rate, or cost of capital, were 10%, we would prefer Project A on an NPV basis, but still prefer B on the basis of the IRR. Check the numbers!

This highlights two problems with the NPV method that has not been mentioned so far: the NPV is sensitive to changes in the discount rate, so much so that in certain cases, a change in the rate, say from 14% to 15%, can yield a change in choice. Also, the project with the steeper curve (Project A) will be more sensitive to changes in discount rates than the other – in terms of its NPV.

1. Many thanks to Mr. Yeshaya Gross (Touro College, class of 2022) for creating this diagram.

2.5 The Solution Explained: A Retrospective Bird's Eye View

The discount rate, based on the company's cost of capital, may be somewhat arbitrary as it is subject to fluctuating market conditions. If the crossover point is 14.53% (as above) and the firm's discount rate cost of capital is either 14.4% or 14.6%, we would technically favor one or the other project, but just barely. As such, the exact location of the crossover point is, possibly, of great interest. This crossover point "phenomenon" will occur when we are faced with competing projects whose idiosyncratic comparative NPV profiles emanate from radically different cash flow "complexions" – as we saw in the example just completed.

So… what do we already know, and how will it help us in calculating the crossover point? We know that the IRR is the point at which the NPV = 0 for any given project. That is useful.

The fictitious "Project C" represents the difference in the two projects' respective cash flows. Thus, the IRR for "C" would represent the difference between the two projects' NPVs. In other words, $NPV_A - NPV_B = 0$, and IRR_C must will, at this point, equal 0.1453!

2.6 Calculating the MIRR: Negative Interim Outflows

The MIRR may be found by equating the PV of the costs with the terminal values (TV) of the inflows. The TV is calculated, as you will recall, by assuming a future "external" reinvestment rate, which herewith, shall be a "given."

We shall use for this example, a reinvestment rate of 12%. Note that here there were several negative (out-) flows, or costs. In such cases, the negative values are discounted to their respective PVs, and the positive inflows are compounded to their FVs. Then a time value-based return shall be calculated as before, comparing the PV "cost" (in lieu of or as supplement to the original notion of "initial outlay") with the TV. This is the same MIRR formula already discussed. Be careful to use the correct exponents in each instance. We shall illustrate the solution using the cash flows for "Project A" given earlier (see Section #2.2).

Project A

PV of costs =

($300) + ($387 ÷ 1.12^1) + ($193 ÷ 1.12^2) + ($100 ÷ 1.12^3) + ($180 ÷ 1.12^7) =

$952.00

TV of inflows =

($600 × 1.12^3) + ($600 × 1.12^2) + ($850 × 1.12^1) =

$2,547.60

$2,547.60 ÷ $952.00 = 2.6761x

- For every dollar invested, the project will produce $2.67 in the future.
- Remember: the project's life is 7 years.

$MIRR_A = (2.6761)^{1/7} - 1 = 15.1\%$

Summary: The MIRR is calculated by compounding any positive interim cash flows to a Terminal Value, i.e., a Future Value at the term of the project, and comparing it to the Present Value of the negative cash flows – as shown in the formulations above.

2.7 A Word about Discount and Reinvestment Rates

One reason future reinvestment rates may differ from the internal rate of return is that market interest rates simply need not correspond to some corporate project's specific internal rate of return. Moreover, market driven interest rates will change with changing market conditions over time.

Even if market interest rates do not change as time goes on, a normal yield curve would indicate lower reinvestment rates over time, i.e., as the project's horizon gets closer.

Use of the "Spot Rate" curve would enable one to view future short-term reinvestment rates differentially. The Yield-to-Maturity would be the average of the individual rates implied by the Spot Rate Curve.

There are other cogent arguments for not using a single discount rate for the entire range of the project's cash flows. Certain projects are more or less risky than others; use of the firm's overall (weighted average) cost of capital may or may not be appropriate for a given project. Furthermore, the use of the same discount rate for all projects would lead to favoring riskier projects, as their cash flows would be inadequately discounted; the discount rate for riskier projects' cash flows would be higher than it should be if we adjust for risk. The NPV would, therefore, be overstated – *before* adjusting for risk.

Indeed, certain cash flows – *within* a given project – may be more or less risky than other cash flows. Herewith we have not provided a means by which the variable riskiness of numerous cash flows may be differentiated.

2.8 Final Words Regarding IRR versus MIRR

At this juncture, you should be able to state the main points characterizing both the IRR and MIRR. Take note of the following summary points comparing the two methods:

The MIRR…

- Utilizes a *realistic* external reinvestment rate.

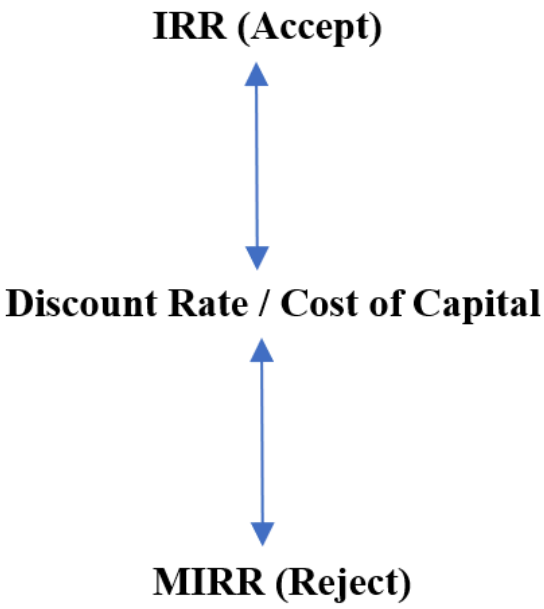
  That's why the MIRR is sometimes called the *External* Rate of Return – ERR as compared to the IRR – *Internal* Rate of Return! Get it?

  - Does not yield any quadratic results – ever.

- Negative interim cash flows are discounted to present value, and positive cash flows are compounded to the horizon. Therefore,

$$(FV / PV)^{1/r} - 1 = MIRR$$

- Does not have the issue of iterations.
- The IRR and MIRR will not yield conflicting rankings / preferences when competitive projects are compared. We will not have a situation where one method says: "accept A," while the other says: "accept B."
- However, the use of an external reinvestment rate, if low enough, may result in an individual project's MIRR being *below* the cost of capital hurdle rate, whereas it may not have been based on the IRR alone.
  - This is because the terminal value will be lower than when calculated using the higher IRR as the compound reinvestment rate.
  - This, of course, means that you may accept a project based on IRR, but reject it based on MIRR – as below.

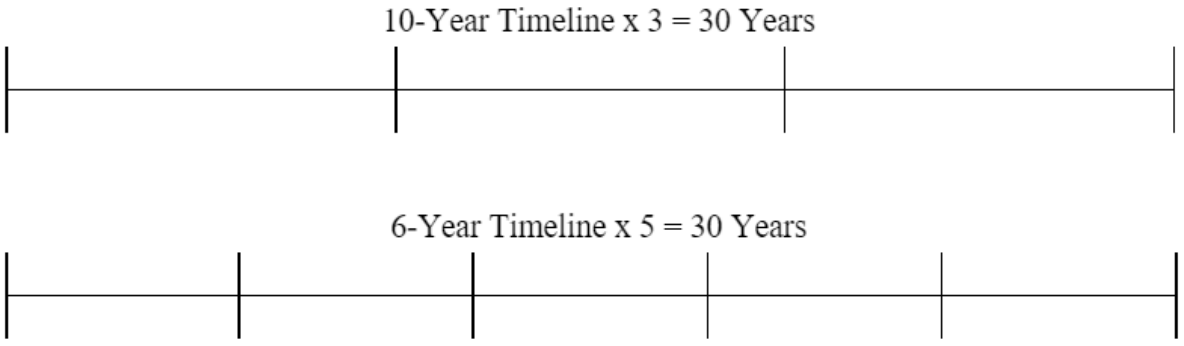

IRR (Accept)
Discount Rate / Cost of Capital
MIRR (Reject)

2.9 Capital Budgeting for Mutually Exclusive Projects with Unequal Lives: Replacement Chain Analysis

Up until now, all the mutually exclusive projects we examined had equal lives; they had equal horizons. What if there are two competing projects whose expected lives are not equal, unlike our earlier examples?

Let us say that one project has a six-year life and another has a ten-year projected life. In order to compare one to the other, we could imagine that as one project's term ends, it is replaced with an identical project with the same life – and same Free Cash Flow projections; we call this "replication" or "replacement." This way, we have equalized the horizons of each project, and we could now compare one to the other based on a Capital Budgeting metric. Let's call this method of analysis the "Replacement Chain Analysis" ("RCA").

In order to complete the RCA in this case, we would have to project out each investment project over a 30-year timeframe in order to equalize the terms of the projects. This is due to thirty-years' being the projects' lowest common denominator. Both can be divided into 30, but nothing smaller than 30 (30 / 6 and 30 / 10). The timelines would be:



If we did not equalize the horizons, we might find that we favor, on an NPV basis, a project simply because of its longer horizon, which provides more cash flows over its greater life.

Let's say Project A has an NPV of $100,000 over its five-year life versus Project B that has an NPV of $110,000 over a twenty-year life. Given what we know about TVM and returns, would it pay to deploy capital for "B" given its minimally higher NPV?

RCA is based on some very restrictive assumptions:

1. That the projects are replicable.
2. That the cash flows used in the first iteration can be used for the replications, i.e., the later iterations.

Let’s try another approach.

2.10 Capital Budgeting for Mutually Exclusive Projects with Unequal Lives: The Annual Annuity Approach

There is a much simpler manner, rather than using the RCA, in which this problem may be resolved. This method goes by three different names: the "Annual Annuity Approach," ("AAA"), the "Equivalent Annual Annuity ("EAA")," or the "Annualized Net Present Value Approach" ("ANPV"). In this method, we first calculate the NPV for each project, given the respective lives. We'll just call it "AAA."

The next step requires calculating the annual annuity cash flow equivalent of the NPV. It is as if one is saying that the project is, instead, an annuity – rather than uneven cash flows as will virtually always be the case – with the same NPV. You will note that this calculation is reminiscent of a mortgage. One simply takes the NPV and divides it by the relevant present value annuity factor. Here, again, is the mortgage formula:

Principal (NPV) = (Annuity Cash Flow) (PVAF)

So, we have two projects with uneven cash flows, and let us say that we have already calculated the respective NPVs, which we shall refer to as "Principal" in the mortgage formula. Please solve for AAA, given the two competing projects' lives and NPVs, and using 12% for the AAA calculation. Take note that, at first glance, you may prefer "B" due to its higher NPV.

	Project A	**Project B**
Life	3 Years	6 years
NPV	$5,000	$6,500
Discount Rate	.12	
Solution: AAA	$5,000/2.4018= **$2,082**	$6,500/4.1114= **$1,581**

In order to arrive at the AAA, we, once again, employed (above) the mortgage formula:

Project A: $5,000 = (x) (2.4018) Project B: $6,500 = (x) (4.1114)

Normally one may assume that the longer the term of the project, the greater the NPV will be. This may often, but not necessarily always, be true. Thus, both methods will be consistent with one another in that only one project will be preferred.

Analysis: In this case, even though "B" has a higher NPV, "A" provides greater annual equivalent free cash flow. "A" is the one to go with – based on AAA. Again, in a manner similar to a mortgage, the "annual equivalent payment" times the PVAF equals the NPV! The project with the higher AAA must, by virtue of mathematical necessity, also have the higher NPV when extended out to a common life. In other words, $5,000 + ($5,000 ÷

1.12^3) = \$8,559 > \$6,500. This solution represents the present value of the project in its first iteration (\$5,000) plus the present value of its replicated NPV in the second iteration (\$5,000 ÷ 1.12^3).

At this point, you may wonder "what use is the RCA approach"? The RCA may be simpler to convey to the managers / decision makers who are not well-informed regarding TVM – even though there will be more number-crunching work in having to come up with more future periods and more cash flows.

2.11 Benefits and Demerits of the Replacement Chain and Annual Annuity Approaches

Here are some summary points to keep in mind when comparing the RCA and AAA approaches.

- The RCA is easier to explain to people who do not understand TVM.
- The AAA requires fewer calculations.
- It may be unrealistic to assume that the projects will be repeated.
- Projects may be renewed, but only once or not at all.
- Revenues and operating costs may need revision in future project iterations.
    - In inflationary circumstances, equipment replacement may be costlier down the road.
    - New technologies may provide operating efficiencies and thus alter free cash flow projections.
- Some (or all?) of the above considerations may be built into the RCA, but not the AAA.
- The AAA views the future cash flows as a “no-growth annuity.” We look at the cash flows, in theory as being equal each year, i.e., as an “Annuity Equivalent.”
- Due to there being unequal lives, it is possible that the NPV and IRR rules of thumb will yield conflicting outcomes for competing projects. (Indeed, we saw this in the example above.) When adjusting for *equal* lives using the RCA, the conflict will disappear.
- In practice, it may be best to project out future Free Cash Flows for extended terms and to calculate the RCAs. Maybe.

2.12 Sample Problem: NPV and AAA for Unequal Lives

Given:

- WACC (weighted average cost of capital; discount rate) = 0.08
- Cash flows projections as below – on the next page
- The projects are mutually exclusive
- Note: the projects have different expected lives

To Solve:

- Calculate the NPV for both projects, using the given unequal lives.
- Re-calculate the NPV for Project B, assuming that it is replicable as initially projected.
- Calculate the AAA for both projects.
- Based on the numbers calculated, which project would you choose?

Note:

This example is derived from: http://www.investopedia.com/exam-guide/cfa-level-1/corporate-finance/comparing-projects-unequal-lives.asp. Thanks to Chaim Halberstam and Ariel Dorfman (The Lander College for Men class of 2019) for re-working the numbers to suit pedagogical requirements.

2.13 NPV and AAA for Unequal Lives (Worksheet)

($ Millions)

	Machine A		Machine B	
Year	**Cash Flows**	**Discounted Cash Flow**	**Cash Flows**	**Discounted Cash Flow**
0	($10,000)		($5,000)	
1	2,000		1,750	
2	3,500		3,250	
3	3,250		3,000	

NPV		

3			($5,000)	
4	3,000		1,750	
5	2,750		3,250	
6	2,500		3,000	
NPV				
AAA				

2.14 NPV and AAA for Unequal Lives (Solutions)

Solution:

($ Millions)

	Machine A		Machine B	
Year	**Cash Flows**	**Discounted Cash Flow**	**Cash Flows**	**Discounted Cash Flow**
0	($10,000)	($10,000)	($5,000)	($5,000)
1	2,000	1,852	1,750	1,620
2	3,500	3,001	3,250	2,786
3	3,250	2,580	3,000	2,381

NPV		**1,787**

3			($5,000)	(3,969)
4	3,000	2,205	1,750	1,286
5	2,750	1,872	3,250	2,212
6	2,500	1,575	3,000	1,891
NPV		**3,085**		**3,207**
AAA		**667.33**		**693.72**

The table below summarizes the data.

	Project A	**Project B – 3 Years**	**Project B – 6 Years**	**Decision Choice**
NPV	3,085	1,787		A
			3,207	B
AAA	667.33		693.72	B

On an NPV basis, project A is favored when compared to project B, if assumed that B is non-replicable. When assuming equal lives, by replicating Project B, both NPV and AAA favor B.

2.15 Topical Practice Problems: Replacement Chain versus AAA (Problems # 1 – 7)

1. Under what circumstances would you use the Replacement Chain and Annual Annuity Approach respectively? What are their relative advantages and disadvantages? Make a list.

---

2. Yur Company has two mutually exclusive investment choices for replacing old machinery. Assume that both projects can be repeated (replicated) and that the relevant Weighted Average Cost of Capital ("WACC"), i.e., discount rate, is 14%.

	Machine A	**Machine B**
Investment Cost	$190,000	$360,000
Expected Life	3 Years	6 Years
Free Cash Flow (Per Year)	$87,000	$98,300

Using both the Replacement Chain and Annual Annuity approaches, upon replacement of the old machine, which new machine should it select – A or B?

---

3. Use the following information for questions # 3 – 5. Show all your math.

- Spread the numbers for the following two mutually exclusive projects:

  a. Both projects have an investment cost of $10,000,000

  b. Project A has an expected life of 2 years after which time it can be replaced at the same investment cost

  c. Project B has an expected life of 4 years

  d. Both projects have average risk; the relevant WACC / discount rate = 10%.

  e. Project A has Free Cash Flow of $6,000,000 and $8,000,000 for years One and Two respectively

  f. Project B has Free Cash Flow of $4,000,000 per year

  g. "Question #3" refers just to "spreading the numbers" based on the "givens."

4. If Project A can be repeated, and using the same assumptions, use the Replacement Chain analysis to determine which project, on an NPV basis should be recommended.

5. Again, using the *Annual Annuity Approach*, which project has the higher annuity equivalent?

Use the following information to answer question # 6. Show all your math.

6. *Yossi's Company* has two mutually exclusive investment choices for replacing old machinery. Assume that both projects can be repeated and that the relevant WACC is 14%.

	Machine A	**Machine B**
Investment Cost	$100,000,000	$132,000,000
Replacement Cost	$105,000,000	NA
Expected Life	5 Years	10 Years
Free Cash Flow	$30,000,000 Per Year	$25,000,000 Per Year

Use both the*Replacement Chain* and *Annual Annuity* approaches.

---

7. Use the following information for the next question (#7). Assume the projects are mutually exclusive. Both projects have an eight-year life. The WACC is 10%. Show all your math.

	Machine A	**Machine B**
Investment Cost	$10,000,000	$15,000,000
Replacement Cost – After 4th Year	$12,000,000	NA
Expected Life	4 Years	8 Years
Free Cash Flow – First 4 Years	$4,000,000	$3,500,000
Free Cash Flow- Latter 4 Years	$4,200,000	Same

Which machine should be selected?

Problem #8 follows in Section 2.20.

2.16 Solution for "Question #2"

Here, once again, is question #2, its solution, and some comments.

2. *Yur Company* has two mutually exclusive investment choices for replacing old machinery. Assume that both projects can be repeated and that the relevant WACC, cost of capital discount rate, is 14%.

	Machine A	**Machine B**
Investment Cost	$190,000	$360,000
Expected Life	3 Years	6 Years
Free Cash Flow (Per Year)	$87,000	$98,300

Using both the Replacement Chain and Annual Annuity approaches, should the firm replace the old machine and, if so, which machine should it select?

Under AAA it may appear that "B" should be selected; its annual cash flow is higher and its overall cost is about the same. In calculating RCA, using the same assumptions for "Machine A" iteratively, we should favor B. Let's see how that works out. First, let's draw a timeline.

(190,000) + $87,000

RCA	0	1	2	3	4	5	6
A	($190,000)	0	0	(190,000) + $87,000	0	0	0
		$87,000	$87,000		$87,000	$87,000	$87,000
B	($360,000)	$98,300	$98,300	$98,300	$98,300	$98,300	$98,300

NPV_A = [($87,000) (2.3216) – 190,000] + [(87,000) (2.3216) – 190,000] ÷ [1.14^3]

= **$20,064.81**

(Where $PVAF_{14\%, 3 \text{ periods}} = 2.3216$)

Another solution: NPV_A = [($87,000) (3.8887) – 190,000] – [190,000 ÷ 1.14^3]

(Where $PVAF_{14\%, 6 \text{ periods}} = 3.8887$)

NPV_B = [($98,300) (3.8887) – (360,000]

= **$22,259.21**

(Where PVAF $_{14\%,\ 6\ \text{periods}}$ = 3.8887)

We see that, using “RCA / NPV” analysis Machine B is favored. Of course, there is no assurance that the original assumptions for “A” will repeat themselves upon project renewal. Changing assumptions could change the choice.

2.17 "Question #2" – Continued

In doing AAA, we must compare Machine A to B in terms of its annual annuity equivalent. Initially, we do not assume that Machine A will be replicated at its term – after three years.

Machine A:

NPV_A = ($87,000) (2.3216) – $190,000 = $11,979

With replication: NPV_A = $11,979 + [(11,979 ÷ 1.14^3)] = $20,065

AAA_A = $11,979 / 2.3216 = $5,159.89

AAA_A = $20,065 / (3.8887) = $5,159.89

Machine B:

NPV_B = ($98,300 (3.8887) – $360,000 = $22,259

AAA_B = $22,259 / 3.8887 = $5,724.07

Conclusions:

- We assume that Machine A may be replicated at its term for another three years. Since its annuity equivalent cash flow (AAA) is less than B's, we choose the latter.
- The RCA also favors "B" since its NPV is greater than A's. (This assumes that "A" is replicated.)
- Both approaches are, and should be, consistent with one another; the rankings or preferences are the same.
- The NPV and AAA methods are consistent with one another; they agree. Since the AAA uses the NPV as its present value, the AAA annuity will also be higher.

2.18 Solutions for "Questions #3-5"

($)

	Project A		Project B	
Year	**Cash Flows**	**Discounted Cash Flow**	**Cash Flows**	**Discounted Cash Flow**
0	(10,000,000)	(10,000,000)	(10,000,000)	(10,000,000)
1	6,000,000	5,454,6000	4,000,000	3,636,400
2	8,000,000	6,611,200	4,000,000	3,305,600
NPV		**$2,065,800**		
2	(10,000,000)	($8,264,000)		
3	6,000,000	4,507,800	4,000,000	3,005,200
4	8,000,000	5,464,000	4,00,000	2,732,000
NPV		**3,773,072**		**2,679,200**
AAA		**1,190,234**		**845,326**

The NPV for Project A – when replicated – was calculated as follows: \$2,065,800 + (2,065,800 ÷ 1.1^2) = \$3,773,072. If one had just added up the numbers in the column, the sum would be: \$3,773,600. The discounted cash flow numbers were calculated by using interest rate tables.

See if you can find alternate solutions to this problem. One alternate would involve using annuity factors. Since the cash flows for Project B are equal, the NPV solution could have been calculated using the annuity approach, rather than the "long" approach utilized above.[1]

When assuming equal lives, by replicating Project A, both NPV and AAA favor A. Note that "project scale" is not at issue here.

1. Many thanks to Chaim Halberstam (The Lander College for Men class of 2019) for providing the solution to this problem.

2.19 Solution for "Questions #6 & #7"

Question #6

	NPV Calculation	Machine A
Cost =	$100,000	($100,000)
PV of CF		
(Stage 1)	$30,000 x 3.4331=	102,993
PV of Cost (Stage 2)	$105,000 x 0.5194=	(54,537)
PV of CF (Stage 2)	$102,993 x 0.5194=	53,495
	NPV=	**$1,951**

AAA

1,951 / 5.2161= AAA = **$373.32**

Notes:

1. Can you think of a slightly shorter way in which to calculate the NPV for "A"? Hint: We have a $30 million annuity for ten years.
2. There really is no point in calculating the AAA. Because the machine's anticipated replacement cost has been changed, the AAA for Stage 1 and the combined AAA for both stages will be different. In comparing the two machines' respective AAAs (see below), the PVAF denominator is the same in each calculation therefore, of course A will be preferred, as its NPV/numerator is greater.

	NPV Calculation	Machine B
Cost =	$132,000	($132,000)
PV of CF	$25,000 X 5.2161	130,403
	NPV=	**($1,597)**

AAA

(1,597) / 5.2161 = AAA= **($306.87)**

Recommendation: While Machine A is not terribly attractive, we would prefer it over B. The corporation may choose not to replace it after the first five years, because the second five years are projected at a loss.

Question #7

A: -$10,000 + 4,000 (3.1699) – 12,000 (0.6830) + 4,200 (3.1699) (0.6830) = $3,576

B: -15,000 + 3,500 (5.3349) = $3,672

Recommendation: Choose B.

Note: In this case, both the machine's anticipated replacement cost and the cash flows were changed, not "replicated."

2.20 NPV and AAA One Last Problem ("Question #8")

-You are given two competing projects, “A” and “B.”

-Project A costs $1,000 and produces a $250 annuity for twelve years.

-Project B costs $150 and produces a $100 annuity for four years. Analysts imagine that the project can be replicated at least three times with no change in the -projected data.

-K = 10%, annual

1. $NPV_A = 250\ (PVAF_{0.10;\ 12}) – 1{,}000$

   $= 250\ (6.8137) – 1{,}000$

   = **$703.425**

2. $NPV_B = (100)\ (PVAF_{0.10;\ 4})$

   $= (100)\ (3.1699) – 150$

   = $166.99

   $= \$166.99 \div (1.10)^0 = 166.99$

   $+ \$166.99 \div (1.10)^4 = 114.05$

   $+ \$166.99 \div (1.10)^8 = 77.90$

   NPV_B = **$358.94**

Alternate Solution: $166.99 ÷ 3.1699 = 52.68 (This is the AAA)

52.68 (6.8137) **= $358.94**

Another Solution: $100 (6.8137) – 150 – 150 (0.6830) – 150 (0.4665)

= $358.94

3. AAA_A =

   703.425 = (x) (6.8137)

X = **$103.24**

4. AAA_B =

<u>One way</u>: $358.94 = (x) (6.8137) = **$52.68**

<u>Another</u>: $166.99 = (x) (3.1699) = **$52.68**

5. We choose Project A

	NPV	**AAA**
A	$703.425	$103.24
B	$358.94	$52.68

Remember: AAA does not require replication, assuming no change in projected data for subsequent periods.

2.21 The Capital Budgeting Process Summary and Review

The corporation will/should go through the following steps prior to making a capital allocation decision.

- Identify and collect the relevant data from the field. Include expected changes in every affected functional area of the business.
- Use *Free Cash Flow* numbers.
- Do not use accounting-type data. Rules are made by people; rules are arbitrary. It is cash that creates value.
- Use incremental numbers only. It is these data upon which we base our decisions.
- Organize your data assiduously. Decide on how many future periods you will project out. Figuring projected *EBITDA* and *Free Cash Flow* is not so simple.
- Account for any *operating efficiencies* or *economies of scale* that are generated by the investment.
- Expected inflation should be built into the projected cash flow data. The projected data will be discounted using the *(weighted average-) cost of capital*.
- Financing flows are ignored in arriving at *Free Cash Flow*; they are part of the discount rate (weighted average-) cost of capital.
- Choose the correct cost of capital. It changes periodically. We observed that small differences in discount rate can matter.
- Consider the competition. What will happen competitively if the firm rejects the project?
- What capital budgeting technique will you choose?
- Are you brave (or foolish?) enough to go against the capital budgeting decision rules based on, potentially subjective, qualitative notions?
- Some companies may choose not to make certain investments for ethical reasons – even though the project may be hugely profitable. These companies will refuse to pollute, for example. “ESG” or Environmental and Social Governance considerations are important today to many corporations.

2.22 Spot Rates

The reinvestment rate for the MIRR is exogenous or external; so far for us, in other words, it was a "given." The analyst must project the project's reinvestment rate, based on economic and financial matters about which he is aware within the corporation's business environment. Absent such information, s/he must, obviously, look elsewhere.

Some useful clues may be garnered from the bond yield curve, which embeds critical information about future short-term interest rates. The "spot curve," which we delineate below, will provide not merely a single future reinvestment rate, but multiple and varying rates over the term of the project. The provision of varying reinvestment rates will satisfy the claim that future rates cannot be monolithic.

Example I:

1.5-Year Treasury Bond

Price = 99.45 (Par = 100)

Coupon = .085

6-Month Zero YTM = .08

1-Year Zero YTM = .083

- What is the theoretical 1.5-year Spot Rate (yB_3)?

Solution:

Coupon Payments = ($100) (.085) (.5) = $4.25

$$99.45 = \frac{4.25}{(1+y_1)^1} + \frac{4.25}{(1+y_2)^2} + \frac{104.25}{(1+y_3)^3}$$

Where y_1 = (.080) (.5)

Where y_2 = (.083) (.5)

Where y_3 = ?

$$99.45 = \frac{4.25}{(1.04)^1} + \frac{4.25}{(1.0415)^2} + \frac{104.25}{(1+y_3)^3}$$

y3 = .04465

BEY3 = (2) (.04465) = .0893

Where BEY = Bond Equivalent Yield

Example II:

2.0 Year Treasury Bond

Price = 99.64 (Par = 100)

Coupon = .090

Spot Rates for y_1 – y_3 as in Example #1 above

- What is the theoretical 2.0-year Spot Rate (y_4)?

Solution:

$$99.64 = \frac{4.50}{(1.04)^1} + \frac{4.50}{(1.0415)^2} + \frac{4.50}{(1.04465)^3} + \frac{104.25}{(1+y_4)^4}$$

y4 = .046235

BEYy4 = (2) (.046235) = .09247

Summary:

Maturity	Coupon	Price	Y-T-M	Spot Rates
6- months	0.00%	96.15	.0800	.08000
1-year	0.000	92.19	.0830	.08300
1.5 -years	0.085	99.45	.0890	.08930 (As calculated)
2.0- years	0.090	99.64	.0920	.09247 (As calculated)

Can you graph this?

- Show the terms-to-maturity on the horizontal.
- Show the yields for the two curves on the vertical.
- Note that, in this case, the curves diverge after one-year.

2.23 Comparison of Single Discounted Rate (YTM) and Spot Curve

- You have noted above that the spot curve discounts each of the bond's cash flows at a different rate.
- Alternatively, the bond's YTM applies the same rate to each of the bond's cash flows.
- In the end, using either method will provide the same PV.

Exercise: Using the data for the two-year bond on the prior page, calculate, on a period-by-period basis, the present value of the cash flows and hence the bond's price, using alternately, the single Y-T-M discount rate, and the spot rates. Use the template provided below.

Period	CF	YTM	PV-CF	Spot Rates	Spot Discount Rates	PV-CF
1	4.50					
2	4.50					
3	4.50					
4	104.50					
	118.00					

2.24 Comparison of Single Discount Rate (YTM) and Spot Curve (Solution)

Period	CF	YTM	PV-CF	Spot Rates	Spot Discount Rates	PV-CF
1	4.50	$(1/1.046)^1$	4.30	.0400	$(1/1.0400)^1$	4.33
2	4.50	$(1/1.046)^2$	4.11	.0415	$(1/1.0415)^2$	4.15
3	4.50	$(1/1.046)^3$	3.93	.0447	$(1/1.0447)^3$	3.94
4	104.50	$(1/1.046)^4$	87.30	.0462	$(1/1.0462)^4$	87.22
	118.00		**99.64**			99.64

- While the discount rates vary for each period, the two solutions yield the same dollar price.
- A bond may be thought of as a “portfolio” of (interest-only and principal-only) zero coupon bonds
- US Treasury Strips provide a direct view of the spot curve. Strips and coupons are priced differently – they have different yields.
- Strips may be created by breaking down a coupon bonds coupons and principal payments and re-packaging them as zeros.
- The YTM is the average of the spot rates.
- Some say that the YTM is derived from the spot-curve, i.e., only after the bond’s dollar value is known
- Although we do not present it herewith, the analyst may choose to add a “risk premium” of a certain percent to, or across, the spot curve to reflect the added risk over the bond yield curve embodied by the subject investment project. (“Risk premia” shall be a topic of discussion in the “Component Capital Costs / The Capital Asset Pricing Model” section following immediately below.)

2.25 Market Value Added (MVA) and Economic Value Added (EVA)

Accounting data do not usually reflect current market values; rather, they reflect historical costs (and other distortions). This makes it difficult to make assessments of corporate performance. *Market Value Added* (MVA) delineates the difference between the firm's equity's market value and its book value as recorded in its balance sheet. This difference is intended to gauge management's performance and address shareholder concerns.

For instance, if a firm issued equity at $10 per share and the price/market value is now $15 per share, the corporation has, presumably, performed well. The $5 increase may be multiplied by the number of shares in order to measure the increase in firm value realized over the time period during which the shares were originally issued. Put differently, this difference represents the increase, after retaining and accumulating historical earnings, to which the shareholders arguably are entitled, i.e., the difference between the shareholders' initial contribution to the firm and its current value. While this may be useful for evaluating market performance, it is not very helpful in gauging management's performance. Part of the increase in share value may be attributable to secular market increases rather than management performance. For example, the stock's price may have gone up simply because interest rates have decreased. Another assessment tool is "Economic Value Added."

Economic Value Added (EVA), a.k.a., "economic profit" is calculated as follows:

EVA = NOPAT – Annual Dollar cost of capital

Where,

NOPAT = net operating profit after taxes = EBIT (1 – T) and

Annual Dollar Cost of Capital = (total investor supplied operating capital) (After-tax percentage cost of capital)

Alternatively, EVA can be calculated in another way, i.e., a way that conveys a different meaning and perspective.

EVA = (Equity Capital) (ROE – Cost of Equity Capital)

Take note that "Equity Capital" is expressed in dollar terms while ROE and Cost of Equity Capital are expressed in percentage terms.

These formulas imply that firms can increase EVA by investing in internal projects that provide returns in excess of their cost of capital.

(Note that cost of capital, in effect, means the returns the firm would earn on alternative choices of equal risk. The management must produce a retrurn equal to the opportunit set facing shareholders.)

EVA estimates the firm's true yearly economic profit and differs therefore markedly from accounting net income,

which ignores the cost of equity. If EVA > 0, the firm's management is thought to have contributed positively to its growth.

Given the foregoing, we may rewrite the firm's market value as:

$$\text{Equity Market Value} = \text{Book Value} + \text{PV of all future EVAs}$$

2.26 Summary: Capital Budgeting

Capital budgeting has to do with the allocation of scarce capital resources to the firm's most profitable potential investment projects. This assumes that corporations are in business to maximize profits and/or cash flows. As a starting point of the analysis, we utilized operating cash flows, which were, presumably, provided ex-ante by an analyst's projection, a subject, which we covered earlier.

We next compared several capital budgeting techniques, assuming the projections given were certain, an unrealistic assumption indeed. In order of discussion, we looked into:

- Payback
- Discounted Payback
- Net Present Value
- Profitability Index
- Internal Rate of Return
- External or Modified Internal Rate of Return
- Annual Annuity Approach
- Replacement Chain Analysis
- (Market / Economic Value Added)

Numerous faults were cited with each alternative approach, and no method was found to be perfect. In the end, the analyst will have to cope not only with the imperfections of capital budgeting techniques and the uncertainty of projections, but qualitative issues that may impact the data in immeasurable ways. The author, in his experience, has always used both NPV for a dollar value and MIRR for a return figure in his analyses.

We also discovered that it is possible, indeed reasonable, to use more than one discount rate; such rates may be derived from the Spot Curve and, at times, by using Forward Rates.

The highest use of capital is not to make more money,

but to make money do more for the betterment of life.

-Henry Ford

2.27 Review Questions: Chapter Two

Review Questions: Chapter Two

1. Assuming a 4% Reinvestment Rate, what is the project's Modified Internal Rate of Return? (Yes … I know. Another MIRR problem!)

Initial Cost= $22,545

CF_1= $7,500	CF_5= $4,750
CF_2= $6,275	CF_6= $4,650
CF_3= $4,000	CF_7= ($2,150)
CF_4= $4,850	CF_8 = $0

2. In this chapter, we observed a problem in which one or another project was favored – on the basis of its NPV, depending on whether discount rates were "low or high." In you own *words* and in writing, restate the problem, and discuss how it was resolved in detail.

3. How is it possible, if at all, that the IRR says accept two projects, while the MIRR says accept only one of the two? Aren't IRR and MIRR supposed to be consistent with one another?

4. How does the Spot Curve resolve the issue, if it indeed does at all, of probable multiple future rates?

5. In detail, explain how a firm's Limited Capital may direct it to accept projects that it might not rank as highly as others – as opposed to the situation where capital were not at issue.

6. Given the following, calculate the Replacement Chain Net Present Value and Annual Annuity Equivalent for each project. You may assume Replication. Use 2% as the Discount Rate. Which project will you favor?

Project A	**Project B**
Initial Cost = $22,545	Initial Cost = $1,254
CF_1 = $7,500	CF_1= $8,750
CF_2= $6,275	CF_2= $7,650
CF_3= $4,000	CF_2= $7,650
CF_4= 4,850	CF_3= ($120)
CF_5= $4,250	CF_4 = $0000
CF_6= $4,000	
CF_7= $0000	

7. Do you think that Scale is an issue in the prior question?

8. You are given the following observed annual Market Yields. Calculate the Spot Curve.

$$YTM_1 = 0.0100$$

$$YTM_2 = 0.0150$$

$$YTM_3 = 0.0200$$

$$YTM_4 = 0.0225$$

$$YTM_5 = 0.0300$$

$$YTM_6 = 0.0310$$

Chapter 3: The Cost of Capital: Component- and Weighted-Average Capital Costs

3.1 Chapter Three Learning Outcomes

Learning Outcomes

In this chapter, you will:

- **Learn** that *Corporate Capital Costs* and *Investors' Returns* are the same.
- **Explore** each of four Capital Components and their respective cost rankings.
- **Provide** the individual Capital Components' respective costs.
- **Define** and Calculate the *Weighted-Average Cost of Capital* (WACC).
- **Identify** the various factors that will affect a firm's "WACC" and the quantitative effects of the factors.
- **Introduce** how a project's Marginal Cost of Capital is utilized in decision-making.
- **Achieve** a high degree of expertise with the *Capital Asset Pricing Model*.

3.2 What is the Discount Rate Anyway?

In our discussions regarding Capital Budgeting Methods, we have, so far, assumed a given Discount Rate, "R." It was provided without explanation. In this chapter, we will learn how to calculate R and how to apply it. We will also examine some nuances.

First, let's recall how R was used in Capital Budgeting. In each of the following methods, it was used as a *Discount Rate*:

- Discounted Payback Method
- Net Present Value
- Profitability Index
- Replacement Chain Analysis
- Annual Annuity Approach

You will note that the Simple Payback Method is absent from this list.

Next, in the following methods, R was used as a *Hurdle Rate*.

- Internal Rate of Return
- Modified Internal Rate of Return

The rule of thumb for the two methods above require that the calculated rate must exceed the hurdle rate in order for an independent project to be "accepted."

To calculate the Discount Rate, we will first identify the four Capital Components which a firm may acquire so that it may finance the acquisition and maintenance of its assets:

- Debt
- Preferred Equity
- Common Equity
- Retained Earnings

You will note that only Retained Earnings is an internal capital source; the other components must be obtained externally. Each component will have its own economic cost, which will be calculated. Once that is done, we will calculate the Weighted Average Cost of Capital – "WACC" – for all four components combined. The WACC will serve as our discount and hurdle rates.

Finally, you will recall that the Free Cash Flow projections we imported into our Capital Budgeting templates excluded capital costs. Here – finally – we utilize it, not as a dollar amount, but as a rate.

3.3 The After-Tax Cost of Debt Capital

Interest on Debt is legally required to be paid first – before any dividends are paid on stock. It is also legally required to be paid. If the company does not pay its interest in full and on time, it has "defaulted" in its loan obligations; it has violated a legal contract called a "bond indenture."

Moreover, we must consider the fact that bond interest, i.e., "interest expense," is tax-deductible. Let's see how this tax-deductibility affects taxes and company earnings by way of the following example.

Given: In each of the following two scenarios assume that:

$$EBIT = \$500,000$$

$$T = .40$$

Scenario I: No debt, no interest expense

Scenario II: Debt of $1,000,000 (Assume a par bond where i = YTM)

$i = .10$ (i = coupon rate of 10% per annum)

Partial Income Statement		**Scenario 1**	**Scenario 2**	***Differences***
Operating Earnings: Earnings Before Interest and Taxes	**EBIT**	$500,000	$500,000	0
Interest Expense	**I**	000	(100,000)	100,000
Earnings Before Taxes (after interest)	**EBT**	$500,00	$400,000	(100,000)
Tax Expense	**T**	(200,00)	(160,000)	**40,000**
Net Income	**NI**	$300,000	$240,000	**($60,000)**
		$\Delta_{NI\text{-}1\ \&\ NI\text{-}2}$= **$60,000**		

Because of the interest expense in "Scenario 2," the company earned $60,000 less, on an after-tax basis, than it would have under the no debt scenario ("Scenario 1"). That is a net cost of $60,000 versus $1,000,000 in debt, or 6%. This is equivalent to what we would have derived had we used the formula:

$$\boldsymbol{K_d} = i\,(1 - T) = .10\,(1 - .40) = \mathbf{.06}$$

Taxes paid are reduced by $40,000 due to the lower taxable income (EBT). The lower EBT results in a reduction in net income of just $60,000 rather than the full cost of $100,000. (i.e., $100,000 in interest paid less tax savings of $40,000 = net income reduction of $60,000).

Hence, .06 is the after-tax cost of (debt) capital on $1,000,000 of debt in this case, or **$60,000**. In practice, we may alternatively use the after-tax cost of debt as herein calculated, or some YTM-related, after-tax percent. Tax deductibility mitigates the cost of debt!

Note:

We have assumed in this example a “Par Bond,” i.e., one that was sold by the corporation for 100% of its Face Value. In the case where the bond was issued above or below Par, we must use the bond’s “Yield-to-Maturity” in order to calculate the expense.

3.4 Flotation Costs

First, a word about “Internal” and “External Funds,” and “Flotation Costs” – before we use these notions on the ensuing pages. We discussed Internal and External Funds earlier, so the reader is referred to the page entitled: “External Funds Needed Formula.” (“CTRL-F” it in order to find it in this text’s PDF form. It is again discussed in depth further below.)

A company needs capital (right-hand side of the balance sheet), or funds, in order to finance its assets (left-hand side). It needs assets in order to produce sales and, hopefully, profits. No assets, no production. Some of the needs for funds will be provided internally.

External funds are those (capital-) funds, which the company must access from “outside” the company’s normal business activities; external funds are acquired in order to supplement any deficiencies that are likely to occur when funds generated from operations are insufficient for fueling growth in sales and profits, which require the use of additional assets. External Funds may include borrowings or new Equity capital. This may occur in the case of a start-up that has sparse cash flow or in the instance of a company experiencing a downturn in its business. Acquiring external funds is not something a company does routinely.

External Funds include borrowed money (i.e., debt), and preferred and common stock. There are economic costs associated with external funds, each of which costs will be reviewed on the next pages formulaically. Internal funds, for the present purposes, will be limited to Retained Earnings, although Accounts Payable are also considered to be internal.

Regardless of the source of the funds, whether internal or external, they all have *economic* costs, not to be defined necessarily as accounting “expense,” although that may also pertain. Thus, interest on debt is an expense in that it reduces net income, but dividends are merely “expenditures,” paid out from net income. Dividends are not expenses. The acquisition of fixed assets is a capitalized expenditure, not an expense. All are nevertheless economic costs.

An economic cost may also be thought of as an “opportunity cost”; for instance, if one owns stock, he will sell it if his return does not meet with the expected return on a reasonable alternative. Thus, external funds have explicit costs, i.e., interest on debt, dividends on stock, but may also have implicit, opportunity costs. Common Stock, for example, has another, a second, opportunity cost – *growth* in dividend and price.

If the investor demands, or “requires,” growth in dividends and price – because of his opportunity set, the firm must deliver it. The return that an investor requires on his investment due to his opportunity cost becomes the firm’s economic cost as well. If the common stock investor is unhappy with his overall return, including growth, s/he will sell the stock or, perhaps worse, vote out the management and directors when the next proxy comes around. If the investor “requires” a certain return, the firm must deliver.

Now, Flotation Costs are the costs associated with paying Investment Bankers, Accountants, Lawyers, and other advisors their fees for assisting the corporation in raising external funds – debt, and preferred and common stock. We will assume that the Flotation Costs for debt and preferred shares are virtually nil, since these capital sources are usually sold to large corporate/institutional investors in sizable allotments. However, as common shares are

usually sold to small investors in small amounts, substantial marketing is involved and therefore more banking fees, or Flotation Costs, pertain. Flotation Costs, again, are the costs associated with “floating” the issue.

There is no cost to retaining earnings; ya just do it! So, internal equity is cheaper than external equity because there are no Flotation Costs. It is within the firm’s normal course of doing business that it keeps some – or all – of its profits in the corporation so that it will have funds for investment in growth opportunities – as opposed to paying them out to shareholders as dividends.

3.5 The Cost of Capital (Summary of all capital components' respective costs)

Note that here we shall use "K," the corporate cost of capital rather than "R" the investor's return. It is the same thing; you may see either notation out there. As we stated above the corporation must meet the investors' expectations of returns ("R"); consequently, K and R are the same thing. The return demanded by the investor is the economic cost ("K") to the corporation.

The After-tax Cost of Debt Capital

$$K_d = i(1 - T)$$

Example:

I = interest expense = 10%

T = Tax Bracket = 40%

K d = After-tax cost of debt = 0.10 (1 – 0.40) = **0.06**

Note: Flotation costs for debt are assumed to be virtually nil and are excluded from the formula. Remember, interest is tax-deductible; it is an expense that reduces taxable income, so the interest cost to the corporation is stated after taxes.

The Cost of Preferred Equity Capital

$$K_p = D_p \div P_{net}$$

For Equities, we shall call upon your knowledge of the Dividend Discount Models.

Example:

D_p = Dividend on the preferred stock = \$2.00

P_{net} = Preferred Stock Price (net of the immaterial Flotation Costs) = \$20.00

K_P = Cost of Preferred Stock = 2 ÷ 20 = **0.10**

Note: You will note that this is a modification of the no-growth perpetuity model.

The Cost of "Internal" Equity (Retained Earnings)

$$k_{RE} = (D_1 \div P_0) + G$$

= Dividend Yield + Growth Rate of the Dividend

Example:

D_1 = Next Year's Dividend = D_0 (1 + G). **Assume D_1 = $1**

G = Dividend Growth Rate = (D_1 ÷ D_0) – 1. **Assume 5%**

P_0 = Current Stock Price. **Assume $10**.

K_{RE} = Cost of (internal) Retained Earnings = (1 ÷ 10) + 0.05 = **0.15**

Notes:

- k_{RE} represents the investors' "opportunity cost." Management should be able to return to investors – in the manner of dividends and/or additions to Retained Earnings – at least as much as the investors could earn on their own.
- You will also note that common equity has two components to its economic costs:
    1. the explicit cost of the dividend payment (if there is indeed any dividend paid), and
    2. the more implicit economic cost of growth in the dividend, and thus growth in the price of the shares (according to the basic notion of the Dividend Discount Model). Both costs are economic costs (rather than accounting "expenses"), while only the dividend involves any cash outflow. Dividends, remember, are not expenses. *Dividend Growth* is an "opportunity cost," as the investor demands it.

The Cost of "External" Equity (Common Stock)

We said that common stock is sold in smaller allotments and therefore has greater Flotation Costs, which we must take into account (while we ignored the flotation costs for Debt and Preferred Stock discussed above due to the costs' immateriality).

Here's how it works. Let's say common stock is offered, for the first time, by a corporation to the public in an "Initial Public Offering" ("IPO") at $10 per share, and that the investment bankers take a fee of $0.50 per share. The company will keep just $9.50 even though the shareholder paid $10. The bankers' fee, or the Flotation Cost ("F" in the formula below) to the issuing corporation is 5%, i.e., $0.50 ÷ $10. Take careful note that "F" in the formula is expressed in percentage terms.

IPO Price	$10.00	100%	
Fee ("F"= 5%)	(0.50)	5%	**F**
Net Proceeds to Company	**$9.50**	95%	**1-F**

In other words, the net proceeds to the company =

$$[(P_0)\ (1 - F)] = [(\$10)\ (1 - 0.05)] = \mathbf{\$9.50}.$$

Using, again, the same model we used for Internal Equity (and the same numbers), and adjusting for "F." we get a modified formulation:

$$K_{CS} = \frac{D_1}{(P_0)(1 - F)} + G$$

Example:

F = Flotation Costs expressed as a percentage of the IPO Price

K_{CS} = Cost of (External) Common Equity =

$$(\$1) \div [(\$10)\ (1 - 0.05)] + 0.05 = \mathbf{0.1552}$$

The Common Shareholder has a claim on both the Internal (Retained Earnings) and External Equity (Common Stock). External Common Equity is more expensive to the corporation (0.1552) because of Flotation Costs, which are absent with Internal Common Equity [($1) ÷ ($10) + 0.05 = 0.1500], i.e., Retained Earnings.

Notice that in comparing costs, debt is the cheapest capital component, followed by preferred, retained earnings, and last, common equity. Debt is paid first and is least risky to the investor (lender). Additionally, interest on debt is also a tax-deductible expense. Preferred gets paid next. Common Equity, including Retained Earnings, is paid last, and is most risky. Common Stock investors share in the profits, i.e., dividends and retained earnings, and thus have the most to gain. "The more risk, the more the reward." External Common Equity is more costly to the firm than are retained earnings due to Flotation Costs.

Capital Component	Cost of Capital
After- Tax Debt	0.0600
Preferred	0.1000
Retained Earnings	0.1500
Common Stock	0.1552
Weighted Avg. Cost Capt'l (***WACC***)	TBD

There are two more things, which we did not yet cover:

1. We must calculate the overall Weighted Average Cost of Capital or "WACC," which is the Capital Budgeting discount rate we have assumed until now.

2. We must also define project risk, which is implicit to the discount rate. We will get to them forthwith.

3.6 Weighted Average Cost of Capital (WACC)

The firm's WACC is equal to the respective weights of each capital component times its (percentage) cost. Here is an example.

XYZ Corp

As of 12.31.xx

(000)

Long-Term Debt (LTD)	$100,000	Corporate Tax Bracket	40%
Preferred Stock	10,000	Interest Rate in Debt	8%
Common Stock @ Par	10,000	Cost of Preferred Stock	9%
Additional Paid-in-Capital	40,000	Cost of Retained Earnings	12%
Retained Earnings	40,000	Cost of Common Stock	14%
Total Equity + LTD	**$200,000**		

On the next page, we present the solution to XYZ Corp's WACC. On that same page, we also ask (and solve) the following additional question: What if the firm increased its common equity capital by $10,000?

One will note from the table above that there is a kind of hierarchy of costs for the various capital components. When one evaluates the cost of capital, one must view it from the point of view of the corporation (and not the investor) that incurs the cost of raising the capital.

You will note that debt is the lowest-cost capital component among those listed herewith, with preferred, internal, and external common equities higher respectively. (We are ignoring current liabilities herewith.) This hierarchy is so simple because interest on bonds must be paid before any cash dividends on preferred and common stock may be paid. Debt is senior to other capital components in the worst-case scenario of bankruptcy/liquidation of the firm. Although of secondary importance, interest is also tax deductible, which lowers its cost to the corporation even more so.

No common dividends may be paid until preferred dividends are paid first and, in the case of Cumulative Preferred stock, all "arrears" (i.e., past dividends not paid) must be paid in full before any common dividends may be paid.

Of course, common shareholders, while holding the riskiest of the firm's securities, get the most upside as they share an interest in all residual profits and may benefit from growth in earnings and distribution of increasing dividends. As opposed to retaining earnings, issuing common shares imposes upon the firm substantial "flotation costs," which add to the cost of common shares versus, in comparison, retaining earnings. In contrast, preferred dividends are usually fixed.

To summarize, debt is cheapest because interest must always be paid in full and timely. Interest is also tax-deductible to the corporation. Preferred dividends must be paid before any dividends are paid on common shares. From the point of view of the corporation, retaining earnings, which would be the property of the

common shareholder, is a cheaper source of capital funds than raising external common equity funds, which entail substantial flotation costs. Please refer back to the "Cost of Capital" summary page above.

Hurdle Rate

Importantly, the firm's "WACC" is the discount rate which is used in calculating capital budgeting projects' discounted paybacks, NPV, PI, RCA, and AAA. It is also the "hurdle" rate, which is used for the IRR and MIRR. While we did not consider it earlier, projects that have high risk (of their projected cash flows) should have higher discount and hurdle rates than other projects' rates. Some additional interesting questions include:

What does the "Optimal Capital Structure" of the firm mean?

There is no universal answer to this question; it will depend on the factors below. However, we must note that the firm's objective will be to reduce its WACC as much as possible without risking bankruptcy; how much will depend on managerial and shareholder risk tolerances.

What (four) factors affect the optimal structure?

1. Management *risk profiles* (Using debt increases the firm's financial risk)
2. Shareholders' *risk profiles* (Investing in a debt-ridden company implies investor risk)
3. Industry characteristics:
    - Magnitude of P, P, & E investment (some industries, e.g., electric utilities, have intense capital investments).
    - Stability of EBIT
        - Consider also the variability longitudinally of the TIE Ratio.
        - "Times Interest Earned" = EBIT ÷ Interest Expense.
4. Relative capital costs in the market. There are times when stocks and bonds relative costs increase or decrease, i.e., the credit spread.[1]

In the end, we say (ridiculously!) that the firm's existing capital structure is ideal; otherwise, rational managers would not have executed it!

- How does the Optimal Capital Structure affect the firm's choice of new financing sources?
- What does the "Marginal Cost of Capital" mean?
    - It is the capital cost of the next dollar financed (see below).

1. For a discussion of Credit Spreads, see Introduction to Financial Analysis by Dr. K. Bigel, Section #13.9 <https://pressbooks.pub/introductiontofinancialanalysis/>

3.7 Solutions to WACC Problems

$WACC_1$ represents the current capital cost. The capital weight for each component is multiplied by its respective cost, and the numbers are aggregated. Here again are the earlier assumptions.

XYZ Corp.
As of 12.31.xx
(000)

Long-Term Debt (LTD)	$100,000	Corporate Tax Bracket	40%
Preferred Stock	10,000	Interest Rate in Debt	8%
Common Stock @ Par	10,000	Cost of Preferred Stock	9%
Additional Paid-in-Capital	40,000	Cost of Retained Earnings	12%
Retained Earnings	40,000	Cost of Common Stock	14%
Total Equity + LTD	**$200,000**		

If the firm increases its equity, the solution will depend on whether the company did so by means of internal ($WACC_{RE}$) or external ($WACC_{C/S}$) equity. The outcomes will differ. The solutions are also provided below.

	$WACC_1$	**$WACC_{RE}$**	**$WACC_{C/S}$**
RE	40	40 + 10	–
C/S	50	–	50 +10
Total Capital	200	210	210

Problem #1:

$WACC_1$ = (100/200) (.08) (1-.40) + (10/200) (.09) + (40/200) (.12) + (50/200) (.14) = **.0875**

Problem #2a: Increase in retained earnings (internal capital)

$WACC_{RE}$ = (100/210) (.08) (.6) + (10/210) (.09) + (**50**/210) (.12) + (50/210) (.14) = **.089**

Problem #2b: Increase in common stock (external capital)

$WACC_{C/S}$ = (100/210) (.08) (.6) + (10/210) (.09) + (40/210) (.12) + (**60**/210) (.14) = **.09**

Obviously, if one uses external, rather than internal, equity, the WACC will be higher.

In either case, the WACC is higher because the firm is increasing its weighting in “expensive” equity. That is to say, the WACC would rise because the component, which was increased in proportion to the total capital, bore a greater cost than the initial WACC.

Should the firm choose to use debt, rather than equity, its WACC, ceteris paribus, would decrease relative to the initial case (problem #1); this WACC would work out to be 8.34%. Its solvency ratios, however, would also deteriorate, and, in a very bad case, the potential resulting reduction in creditability (“default risk”) could cause incremental debt capital costs to increase. (Interestingly, this would result in higher equity costs as well because equity is junior to debt. We will discuss this notable effect below under “The Levered Beta.”).

- What if the company raised capital by issuing more debt?
    - Since debt is cheapest, the WACC would decrease. The percentage of debt relative to total capital would incerase.
- What if the company reduced capital by buying back stock? What would the new WACC be?
    - If the company buys back stock the equity will be reduced. In Problem #1, the percentage of common stock relative to total capital was 60/200 = 30%. Let’s say that the company buys back $30,000 in common stock. Now the new common stock percentage would be 30/170 = 17.6%.

3.8 WACC Practice Problem

You are given the following problem. What is the firm's WACC?

LCM Corp.
As of 12.31.20XX
(000)

Long-Term Debt (LTD)	$250,000	Corporate Tax Bracket	21%
Preferred Stock	50,000	Interest Rate in Debt	5%
Common Stock @ Par	300,000	Cost of Preferred Stock	7%
Additional Paid-in-Capital	10,000	Cost of Retained Earnings	10%
Retained Earnings	500,000	Cost of Common Stock	12%
Total Equity + LTD	**$1,110,000**		

Solution: Here is the formulation – do you agree? YOU do the calculation!

WACC = (250/1,110) (.05) (1-.21) + (50/1,110) (.07) + (500/1,110) (.10) + (310/1,110) (.12)

= ???

Question: What happens to the WACC if the firm raises more debt capital to fuel growth? Assume *ceteris paribus*.

(The answer to the problem above is: WACC = 0.0906. Don't tell anybody!)

3.9 Marginal Cost of Capital

Let's say a firm wishes to grow and has various capital projects it is considering. The *Marginal Cost of Capital* (MCC) is what the firm's financing costs will run for the new project alone. This is not the same as the cost of capital for the firm overall.

We will assume that the firm is facing the following *marginal* component capital costs. Think of "marginal" as "incremental" in the sense that these are the costs of adding to the firm's overall capital in order to invest in a new project. It is only the marginal costs that are relevant to the investment decision, just as it is only marginal profits that relate to the decision.

	MCC
(After-tax cost of) Debt	5%
Internal Common Equity	9%
External Common Equity	15%

If the firm takes on a new project, we will assume, in the following example, that it has decided that it will use 50/50 leverage. Let's further say that the firm will notuse a mix of both internal and external equity – it will use only one or the other equity financing source. It will not use preferred stock. The firm's marginal (weighted average) cost of capital (MCC) could therefore be either of the following:

Financing Choice	**Formulation / Calculation**	**MCC**
Debt + Internal Equity	(0.5) (0.05) + (0.5) (0.09) =	**7%**
Debt + External Equity	(0.5) (0.05) + (0.5) (0.15) =	**10%**

Let's say the project has an initial cost of $4 million. If the firm has $2 million in internal equity, i.e., earnings, which it expects to retain this year, it could borrow another $2 million and finance the new $4 million project with a 7% weighted-average cost of (marginal) capital. If the firm uses external equity capital – either because it does not have the internal equity, because it chooses to pay dividends, or use the capital for other projects – its MCC will be 10%.

If the project requires more than $4 million, and the firm chooses not to, or is unable to, borrow more, its MCC will rise due to obtaining more external equity, which is most expensive. If it uses borrowed money, it is also possible that the cost of debt will rise due to the lenders' demands – should the firm choose to borrow more than it already has.

If the firm chooses to modify its leverage ratio, you may alter the weights accordingly in the template above. As it stands, any independent project that requires up to, but not more than $4 million, would be accepted if its IRR exceeds 7% or 10% respectively – depending on which marginal capital structure management chooses. Keep in mind that the firm will assess the proposed project's incremental Free Cash Flow in comparison with its incremental, or Marginal, Cost of Capital. In other words, the MCC is the project's WACC!

3.10 Changes in the Cost of Capital

What factors may cause the WACC to rise (or fall)? There are both financial and economic causes. What formula or models will you employ to describe the effects of each of the following possible causes?

Financial Causes for the WACC to change

1. A decision on the part of the company's management to alter the *capital structure* from its current proportions. For example, the use of *equity* whether *external* or *internal equity* will raise the WACC, if the company has any debt at all presently.

2. A change in the corporation's *credit rating* in the face of issuing more debt. (Note: This will affect the firm's Beta.)

3. A change in tax laws may affect the after-tax cost of debt. (This will have no effect on other capital component costs.) The formula to apply in this analysis is:

$$K_{Debt} = I\,(1 - T).$$

Economic Causes for the WACC to change

4. Market changes, including:

   a. A change in the bond *yield curve* due to inflation or lack thereof, and Federal Reserve policies and activities. What will that do to the bond's YTM on its marginal debt?

   b. Bond market *Credit Spreads* will fluctuate with macroeconomic conditions. A Credit Spread is the difference in yield between a Treasury and a lower rated bond of equivalent maturity. Credit Spreads tend to widen when the economic outlook worsens and narrow when the picture is more optimistic. What may happen to the firm's marginal YTM – its marginal cost of debt? A firm's marginal YTM will rise when economic conditions worsen.

   c. A change in the CAPM, which provides the required return used by theoreticians as a discount rate. As noted in our discussion below concerning the CAPM, changes may come about in the security market line due to changes in short-term interest rates (R_F) and changes in the Market Risk Premium ($R_M - R_F$), which has a similar impact and cause as do credit spreads.

If the corporation's WACC went down, the NPV on prospective projects would be relatively higher; more projects would be accepted! Similarly, the hurdle rate would be lower and, under both the IRR and MIRR, more projects would be accepted!

3.11 A Word about Linear Equations (Review of Algebra)

Now that we understand how to price, or value, stocks and bonds, we need to turn to the source of the *discount rate* for stocks. (The following model may also be used for bonds, however, we have and will continue to depend in general instead on the market rate, the YTM, for the bond discount rate.) You will recall that the discount rate in the DDM has, so far, been an exogenous variable, i.e., one that is "given" or otherwise imported into the model, or formula, from the "outside," so to speak. We will now turn to the derivation of the DDM's discount rate, the *Capital Asset Pricing Model*.

In order to understand the Capital Asset Pricing Model, or "CAP-M," which provides the discount rate used in the DDM, we must first recall how to utilize basic linear equations.

The standard formulation of a linear equation is:

$$\mathbf{y = a + bx}$$

Where the terms are as follows:

Dependent Variable	y
Independent Variable	x
Vertical Intercept	a
Slope	b
And where	b =Δy /Δx = "rise / run"

We speak of "y" as being the dependent variable because, in a sense, its value depends on everything on the other side of the equal sign. The equation's other independent variable, "x," is exogenous.

You will note that the linear equation has a vertical intercept, "a," i.e., the point at which the line intersects the vertical axis; the intercept may be less than or greater than zero. The intercept is a "constant" rather than a variable.

The slope, "b," represents the steepness of the line. Think of slope in a physical sense, as in the slope of a hill. If the line is steep, it will take several steps up to move just a little distance forward horizontally; one shall need to exert more effort to climb a steep slope. If the slope is relatively flat, for every step one moves "up" along the vertical axis, one makes nearly equal progress forward. The slope of a line is a constant; the relative flatness of it is the same no matter where the slope is measured along its length. Equal effort is required in all locations in order to move forward. Slope is measured by the size of the increment of the movement along the vertical or "y" axis ("rise") versus the equivalent movement along the "x" axis ("run").

We may recognize and equation as linear when it has all the foregoing components. The equation will also be devoid of exponents, which would make the formula non-linear.

3.12 The Capital Asset Pricing Model (CAPM)

The CAPM will provide us with a discount rate ("R" or "k"), which is relative to the amount of "market" (or "systematic") risk a portfolio has. The greater the market/systematic risk, the greater will the return ("R") be according to this model. Further, the model depicts a linear and positive relationship between risk and return.

The model also assumes that investors are "rational" and that the market is "efficient." If investors were not rational, they would not demand *more* return for taking on *more* risk. The risk/return relationship is thus positive. If markets were not, by and large, "efficient," prices would not reflect this rationality; market prices would be random and not reflect accurate "intrinsic values." Intrinsic Value has to do with what the true value of a stock should be.

The return ("R") to the investor will be the discount rate (k), which was used when doing Capital Budgeting. Remember that the *return* to an investor is the *economic cost*, which the corporation must provide. R and K are two sides of the same coin. Once again, Return to the investor is the economic cost to the corporation. The investor expects a certain return which must then be provided by the corporation. If the corporation does not meet the investor's expected or "required" return, s/he will sell the stock and invest in a better alternative, or vote the company's management out. Let's look at the math of this linear model.

The CAPM indicates that a portfolio's "required" or 'expected" return ("R_P") is equal to – in the manner of a linear equation – the risk-free rate of interest ("R_F") plus a "risk premium" ("$R_M - R_F$") as follows:

$$R_P = R_F + (R_M - R_F)\,\beta_P$$

The risk-free rate of return represents an investment that presents no risk. Its Beta would be Zero. That means that the *actual* return will equal the *expected* return. There is no deviation or variability of actual return from expectations. As Beta increases, however, so too does risk, i.e., variability of return, so that the actual return may either *exceed* or *fall short* of its expected return.

Again, this is a linear model and linear equation. At this point, some readers may prefer to go directly to the "Diagram of the CAPM" (below), and then to return **here** for explanation and insight.

The variables in the CAPM equation (above) correspond to the standard linear formula: $y = mx + b$ although the order of the constants and variables is altered for the CAPM. Here is the correspondence of the contents of the two contrasting formulae:

Linear Equation	Definition	CAPM Equivalent	
y	Dependent Variable	R_P	Variable
m	Slope	R_M-R_f	Constant
x	Independent Variable	β_P	Variable
b	Vertical Intercept	R_F	Constant

The CAPM provides a dependent variable, "R_P" (or just R) that represents the investor's "required (portfolio) return." That is to say, given the formula, that a rational investor acting in an efficient market would not accept less

than "R" as his portfolio return given the correlative risk of the investment; to do otherwise would be irrational. S/he therefore requires a return equal to *R.*

One may also think of *R* alternatively as the expected return one anticipates. Again, should the investor's return fall short of his expectations, s/he may sell the stock and purchase another stock that is expected to meet his/her "requirements." S/he can also vote out the firm's management. To "expect" here is to "require."

The required return, *R*, derived from the CAPM, is used as the discount rate, which we import into the dividend discount model. Given the Dividend Discount Model (DDM) and the concept of the Time Value of Money (TVM), as *R*, the discount rate rises, price goes down.

The risk-free rate (R_P) represents the CAPM's vertical intercept. (Try drawing this now, with risk or Beta on the horizontal axis, and R_P along the horizontal, somewhat about the point of origin.) The risk-free asset has, at the risk of sounding redundant, no risk, but will nonetheless provide some positive return.

The risk-free asset is also referred to as the "Zero Beta Asset" simply because there is no (market) risk. As it has no risk, and as the horizontal axis denotes risk and is measured by Beta (while the vertical axis denotes return), R_F will reside on the vertical axis. Zero risk of the horizontal axis will be noted at the diagram's point of origin. Even though there is no risk, the risk-free asset will provide a positive return or else no one would invest in it.

The phrase "Market Risk Premium" ("MRP") refers to the incremental market return (R_M) above the risk-free rate of return (R_M-R_f). This premium corresponds to the market's risk-level, which exceeds zero by a certain amount (MRP on the vertical; risk on the horizontal axis). You may (wish again to) skip ahead and view the CAPM diagram on the following pages.

The MRP is defined as "$R_M - R_F$." Note that we use "R_M," the return on the "market portfolio." If you invest in the market as a whole (e.g., by buying an index fund) or choose an investment whose risk equals that of the market's, you will earn "R_M," i.e., the market return, which is equal to the risk-free rate of return plus the market-risk premium, i.e., $R_F + (R_M - R_F)$.

When the investor chooses to invest in a portfolio other than the "market portfolio" (or in an asset whose riskiness differs from the market's) his/her return will be either greater or less than the market return as defined by the slope of the line. The investor's investment risk will be either to the right or left of the market's risk, and his/her return will then "travel" along the CAPM line – up or down. One last time, the relationship between market risk and return is linear. The line is referred to as the "Security Market Line (SML)."

Beta

You may ask why $R_M - R_F$ does not look mathematically like the slope. Slope should look like $\Delta y / \Delta x$, but here there clearly is no denominator! If you think of $R_M - R_F$ as "Rise," or as the increment along the vertical axis above the risk-free intercept, then what is the value of the denominator? Where is it in the CAPM?

Remember that $R_M - R_F$ is the incremental return, the Market Risk Premium (MRP) awarded the investor for taking on any *risk* (the denominator noted on the horizontal axis) greater than zero – and thus earning a return greater than RF. In order to better understand the denominator, which is the risk to which we refer, let us first turn to the mathematical definition of risk – "Beta."

$$\beta_P = (\sigma_P \div \sigma_M) \times r_{P,M}$$

This formula says that the risk of a security (or portfolio of securities, as the case may be) is equal to the absolute volatility (in statistics, this is called the "standard deviation") of return of the portfolio (σ_P), relative to (divided by) the market volatility (σ_M); we can define "market" as also referring to the "system," as it is often called. If

you are invested in the market portfolio itself (which you can do by buying an index fund), this ratio will be equal to one, i.e., $(\sigma_P \div \sigma_M) = 1$. The numerator and denominator will be the same.

Some securities will have more or less volatility than the overall market and thus greater or less than the market Beta (β_M) on our graph of the linear formula. Beta represents market/systematic risk, or the risk of a portfolio, relative to the "system," the "market." If your portfolio has more than the market risk, σ_M, the ratio will exceed one and vice versa.

Imagine that a portfolio has a standard (or average) deviation (of return) of plus and minus 10%. This means that on some occasions it may return as much as 10% more or less than its mean/expected return. That's volatility! Risk then comes to mean the *extent* to which the return actually earned may differ from expectations (i.e., both better and worse than expected). Imagine also that the market's volatility is 5%. The stock you chose is twice as volatile as the market! However, that is not all.

The ratio of standard deviations $(\sigma_P \div \sigma_M)$ is then multiplied by the correlation coefficient of the portfolio's and the market's return ($r_{P, M}$) in order to place the relationship in a time-sensitive context. In short, correlation serves to pair up both the direction and extent of the portfolio's return movement with the overall market's. If, for instance, the portfolio in question is rising while the market is falling, the said counter-volatility will have the effect of reducing the portfolio's relative risk; the "r" correlation coefficient could then be as low as -1, indicating (perfectly) negative correlation. Beta thus becomes a measure of relative volatility, rather than absolute. Remember, Beta is the only variable in the CAPM.

Getting back to risk as the denominator, if we establish that Beta is a relative measure of risk, and that the portfolio's return is relative to the market return for a given level of portfolio risk, then the market's risk is relative to itself and perfectly correlated with itself – a redundancy, more formally known as an "Identity," or "Truism"! Of course, this must also be technically mathematically true.

$$\beta_M = (\sigma_M \div \sigma_M) \times r_{M, M} = 1$$

If the market Beta is equal to one, then the denominator beneath "$R_M - R_F$," is also one and, as such, need not be explicitly articulated as the denominator to $R_M - R_F$ in the CAPM formula. The "Rise" will be $R_M - R_F$ and the "Run" will be one – Beta minus Zero.

Similarly, if a stock's Beta is greater than one, the stock is relatively more volatile than the market. If Beat is less than one, then it is less volatile than the market. Even if it is less volatile than the market, the stock may still be volatile in an absolute sense, if the market too is quite volatile. Because the world is so small, it is virtually impossible to find securities that are negatively correlated with one another. Correlations thus tend to fall between slightly above zero and one, the latter of which indicates perfect positive correlation.

The definition of "market" is in the eyes of the beholder, or should I say researcher. In reality, the "market portfolio" is defined, most often, as the Standard & Poor's 500 Index, but one may choose any market measure that is suitable.

To summarize, the required return is equal to the risk-free rate plus a risk premium, which is the market return less the risk-free rate modified by the slope of the SML and the investor's choice of Beta. A picture is worth a thousand words, so let's diagram an example (see below).

One more thing first. Recall that the "R" in the CAPM is imported into the Dividend Discount Model as its discount rate. We now see that if the required return goes up (and prices go down), the economic costs to the corporation also go up. Fewer capital projects will be accepted because they are now more costly and thus will not satisfy capital budgeting requisites and expected corporate growth will go down. There will be less capital investment in general.

3.13 Diagram of the CAPM

In order to diagram the "CAPM," let us devise an example. Suppose you are given the following:

$$R_M = .20$$

$$R_F = .05$$

$$\beta_P = 1.5$$

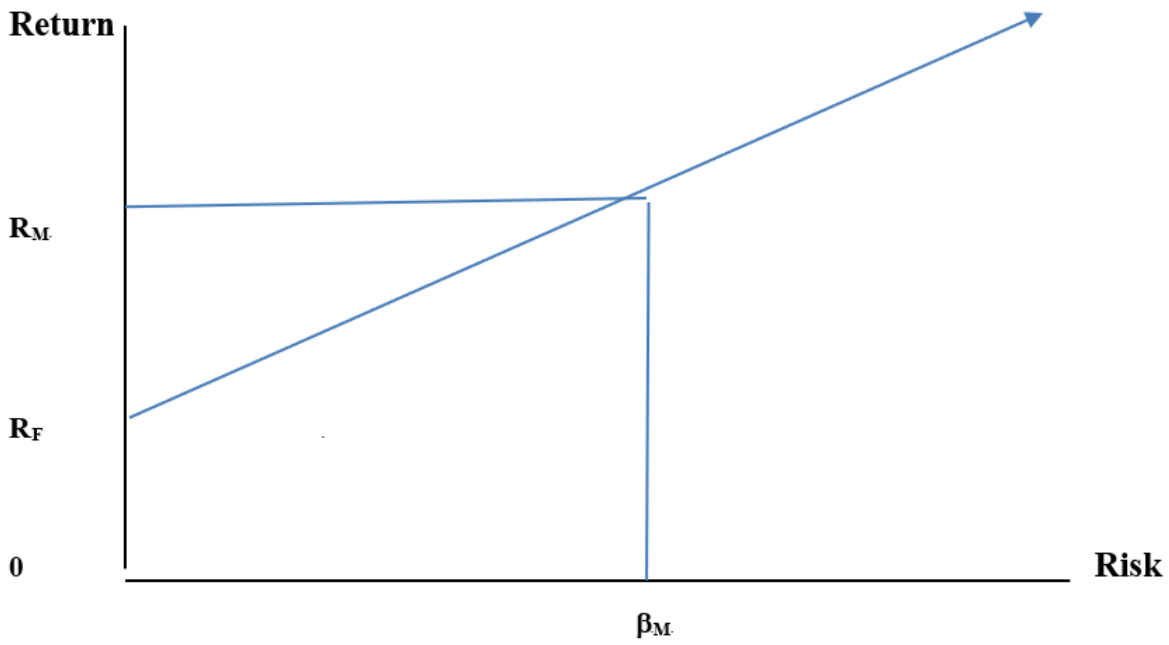
Then,

$$R_P = R_F + (R_M - R_F)\,\beta_P$$

$$R_P = (.05) + (.20 - .05)\,1.5 = .275$$

In this case, you have chosen a stock with greater volatility than the market, and your risk premium will provide a greater return as well. Just beware that risk also means that your expected return is not assured in the short-run and that, in the long-run, things may change so as to alter the formula's inputs. Life is not so simple.

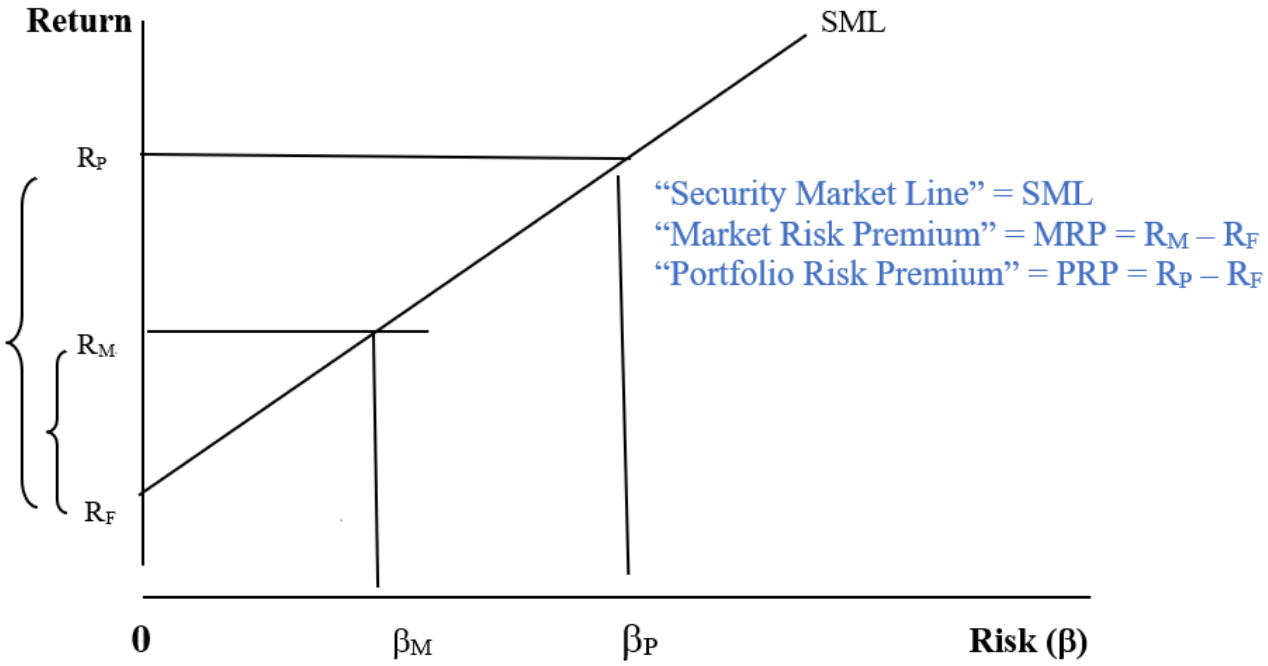
Now we can graph this (try to fill this in):



See the next page for more insight.

3.14 CAPM Diagram

The following completes the diagram on the prior page



$\beta_M = 1.0$
$R_M = .20$
$R_F = .05$
$\beta_P = 1.5$ and
$R_P = 0.275$

Remember: the PRP could be < MRP also.

3.15 Changes in the CAPM

Let's make some simple points here:

- The CAPM provides us with the "Required Return" (R), which we import into the DDM.
- The Security Market Line (SML) is not static, it is "dynamic and will change continually. The "SML" is what we call the line within the CAPM.

This brings up a simple question: What may cause the CAPM to change?

The answers are also pretty simple:

- A change in *inflation*, or in *inflation expectations*, will cause R_F to shift (up or down); the Fed may tighten or loosen in response. When the Fed engages in Open Market Operations, it impacts short-term rates the most, R_F will thus be affected. The SML will shift up or down – in parallel fashion, *ceteris paribus*.
- *Risk premia* will increase when times are bad (recession) and decrease when times are good (prosperity). In bad times, investors are more nervous about the future and the economy; they feel less secure. Thus, for the same degree of risk as before, investors will – later – demand a higher risk premium – above the risk-free rate, to compensate them for the added nervousness.

Increases in the *market risk* premium cause a shift upward in the slope of the SML. Diagrammatically, this means that $R_M - R_F$ will increase when times are bad and vice versa. (A similar phenomenon occurs with bonds – credit spreads[1], i.e., the difference in market yields between investment grade and high yield bonds, increase in bad times; the increase is illustrative of general market nervousness.)

- Both *inflation* and *risk premia* may move.
- While the CAPM may not change, a company's creditability (i.e., default risk) may change leading to a change in its Beta and, hence, in its discount rate (R). This may result from a re-rating of its bonds, or from a change in its capital structure, which, in turn, affects the company's ability to pay dividends and retain earnings. In this case, there is no change in the SML, but the firm's Beta will move to the right or left along the x-axis.

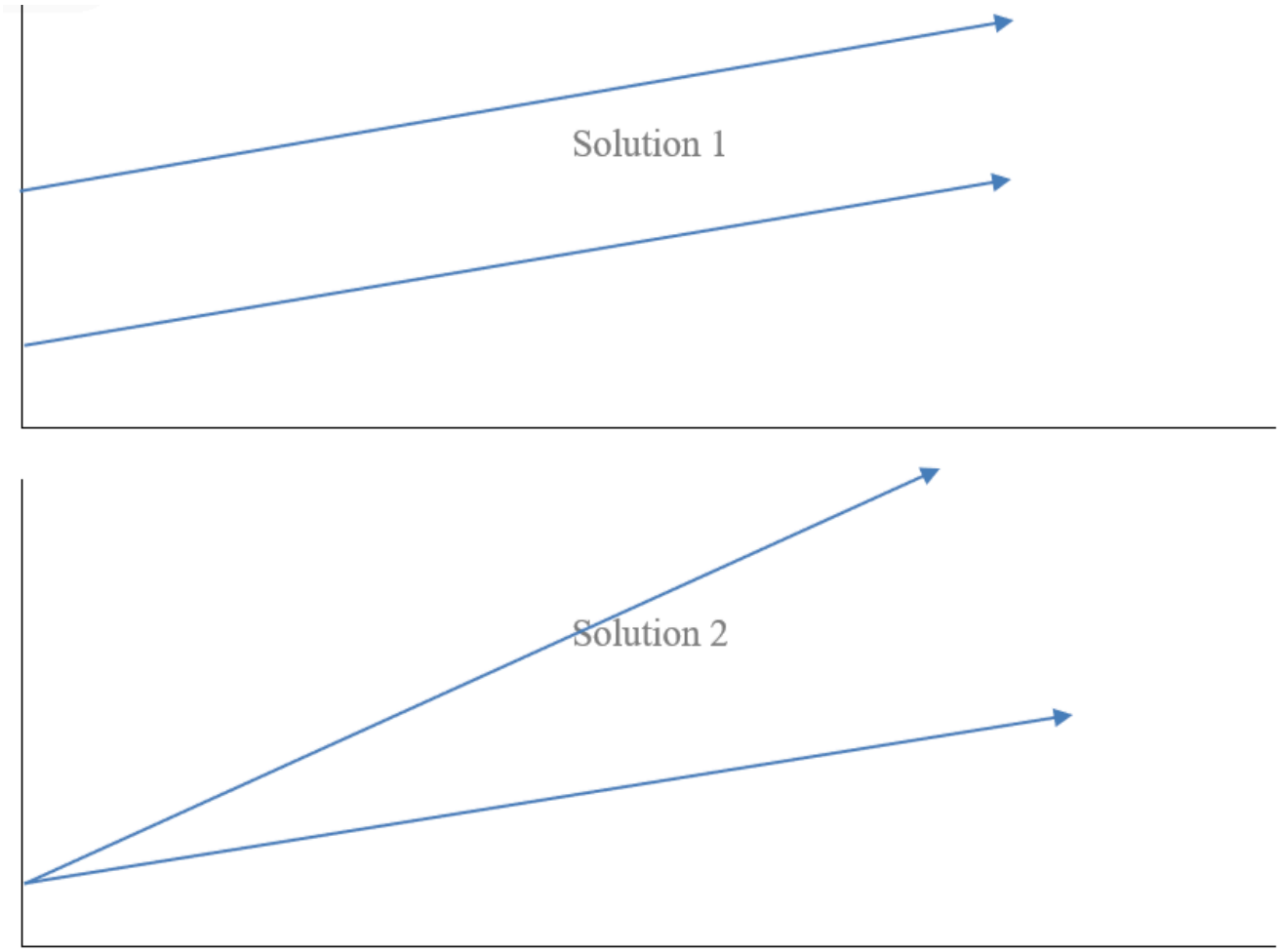
Exercise:

Diagram the axes and the relevant points along each of the axes, and graph the movements described in the three or four (depending on how you count them) bullets above. The diagrams are provided on the next page. Don't look!

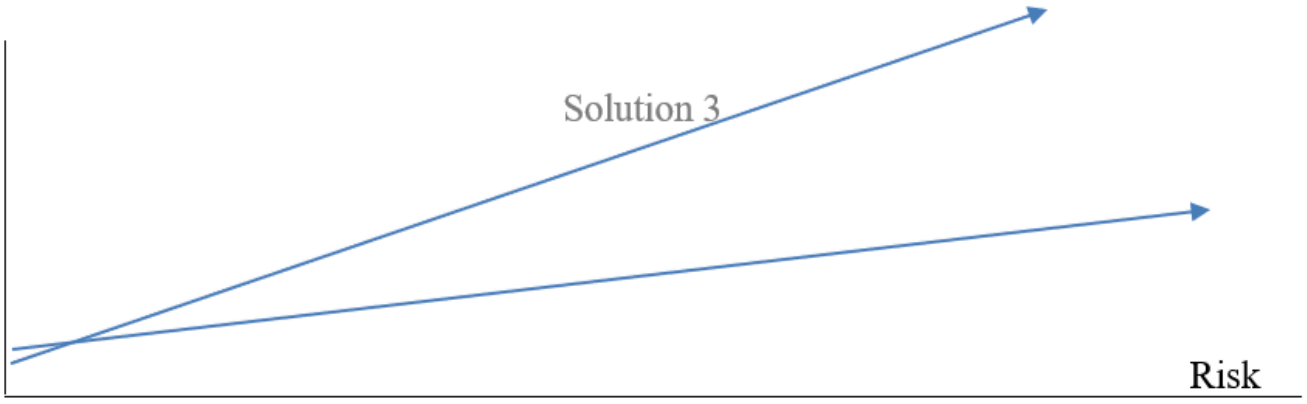
1. For a discussion of Credit Spreads, see Introduction to Financial Analysis by Dr. K. Bigel, Section #13.9 <https://pressbooks.pub/introductiontofinancialanalysis/>

3.16 Diagrammatic Changes in The Security Market Line

Return



Here is one possibility for changes in the two variables. Here, both the risk-free rate AND the slope increased.


Solution 3
Risk

3.17 Accounting Manipulations

Clearly the corporation (i.e., management) has an incentive to do whatever it can in order to reduce its cost of capital, including creating the impression via accounting manipulations that its overall riskiness is less than otherwise may be recognized. There are numerous ways in which this may be achieved. Herewith, we shall examine several.

First, the firm can transform liabilities into off-balance sheet items. Doing so will make the balance sheet appear stronger. Think Enron.

Next, it can "smooth" earnings, by engaging in discretionary decisions as to when to book certain revenues and expenses. One of the most common manipulations has to do with the establishment of reserves for possible asset write-offs. In so doing, a company may show relatively larger net assets, at a point in time, than would be the case if it were more conservatively reflecting certain asset accounts. It also helps smooth earnings.

Specifically, the firm might increase (or decrease) its "Balance Sheet Reserves" in good years. Management will reduce its currently reported profits by expensing additional reserves thus increasing "reserves" held on its Balance Sheets. In bad years, it reduces its Balance Sheet Reserves and inflates its currently reported earnings. These actions will enable it to show more stable operating earnings over time, and thus increase its ability to take on more debt and to service it. Further, if the firm is more profitable, its costs of debt and equity will go down.

Not all of these deceptions are detected even by alert analysts. The benefits of such machinations for the corporation are enormous. For example, a reduction in the annual *Allowance for Doubtful Accounts* of, let's say, $100,000 in artificially reduced expenses, translates into increased Net Income of: ($100,000) (1 – .30) = $70,000.

At a PE ratio of 20 times earnings, this could mean an increase in total market value of (20) ($70,000) = $1,400,000. If this number does not impress you, let's be more realistic for the large corporate world, i.e., "big business." Just add on three zeroes! Now, we're talking!

Corporate managers, who are not often schooled in business and financial ethics, will feel compelled, out of a fiduciary obligation to shareholders, if not out of self-interest, to take advantage of these "opportunities." Let us not forget that managers, whose bonuses may be tied to financial metrics, are frequently also shareholders themselves. The result often is that the company is in fact *more* risky than perceived, to the benefit of the corporation's managers, and to the detriment of bond and equity investors alike.

It is essential that external credit (i.e., bond) and equity analysts, and sophisticated investors, be well-schooled in Financial Statements Analysis. This is real money we're talking about!

3.18 Summary: The Cost of Capital

A firm's cost of capital was shown to be the weighted average of its various component costs. Capital components consist of debt and equity.

If capital component costs rise, the *Weighted Average Cost of Capital* will rise as well. The rise in the WACC will make fewer projects acceptable under our capital budgeting guidelines; each will now have higher capital costs!

To be sure, the subject of valuation cannot be discussed without some treatment of the notion of market efficiency and the Capital Asset Pricing Model. We have assumed that market prices reflect the true, intrinsic values of financial assets, and that the market pricing mechanism indeed works. This is to say that we assume that markets are "efficient." In reality, this subject is far more nuanced than as it has been presented herewith.

Soon, we will review the "external funds needed" model, which is intended to enable the corporation to predict how much external capital is required to support the company's near-term growth plans. Now that we understand this role and the notion of cost of capital in its entirety, we may turn to the issue of capital structure, which addresses the question of how much of the external capital requirement should be raised from debt or equity. Indeed, we shall address the questions of how a firm's capital structure is determined by its management, and what is the ideal singular capital structure (or debt-to-equity ratio)?

3.19 Review Questions: Chapter Three

Review Questions: Chapter Three

1. Calculate the Weighted Average Cost of Capital (WACC) for XYZ Company, given the following data for its four capital components.

Debt	Preferred Stock	Equity Common + Retained Earnings
Debt=$100,000 YTM= 8% Tax Bracket= 35%	Preferred = $50,000 Dividend= $1.50 P= $25	Common Equity = $150,000 Retained Earnings = $75,000 Last Common Dividend- $1.00 Dividend Growh Rate = 2% Flotation Costs = 5% P_0= $12

2. What would happen if the firm issued more debt? Would its WACC increase or decrease. Assume no change in default rate.

3. If the firm issued $25,000 more in debt at 7%, what would its new WACC be?

4. Why might capital components' costs change over time? Provide multiple reasons.

5. What numerous accounting manipulations might the firm employ in order to alter its "reported" WACC?

Solution to question #1:

[(0.08) (1 – 0.35) (100) + (1.5/25) (50) + (1.02/12 + 0.02) (75) + {(1.02) / (12) (1 – 0.05) + 0.02} (150)] / 375 = 0.083167

Chapter 4: Leasing

4.1 Chapter Four Learning Outcomes

Learning Outcomes

- **Predict** the occasional need for “External Financing” by means of the *External Funds Needed Model*.
- **Calculate** the present value of a lease.
- **Evaluate** the impact of *Leasing* on Financial Leverage.
- **Depreciate** and **Amortize** the Lease Asset and Lease Obligation (liability).
- **Explore** the effect of different rates of depreciation and amortization on the firm’s Equity.
- **Consider** the financial benefits and disadvantages of a lease.
- **Recognize** the place of a Financing (“Capital”) Lease within a firm’s overall Capital Structure decision-making.

4.2 External Funds Needed Formula (EFN)

A company needs additional "capital" (i.e., financial resources) in order to grow and to maintain its existing plant and equipment, and to acquire additional inventory. It cannot achieve a sales increase ("growth") without adding on productive "capital assets" (not to mention maintaining existing assets). Thus, increases in Free Cash Flow are necessary to achieve corporate growth and will require investment in numerous assets. Again, in order to grow, the company must invest. Investment creates the capacity to produce more goods for sale and, hopefully, increases both accounting income and Free Cash Flow.

Some of the company's capital needs will be met "spontaneously," i.e., in the normal course of doing business, by retaining some, or all, of its earnings. Such funds may be thought of as having been generated "internally." Some further internal funds will be generated by accounts payable, also in the normal course of business conduct. Payables are, essentially, free, short-term loans provided by the firm's suppliers for, usually 30 days. Payables are thus, financially speaking, free sources of funds. We call these funds, including payables and retained earnings, by numerous names: "spontaneous," automatic," or "internal."

For the balance of its capital requirements, the corporation will need to go outside – to bank lenders, and/or to bond and stock investors for its "external" funds requirements.

Internal Funds:

- Accounts Payable
- Addition to Retained Earnings

We also said that, to the extent that internal funding is insufficient to satisfy all the investment needs, it may seek additional funding externally.

External Funds:

- Short-term bank lines of credit
- Short-term bank loans – "notes"
- Long-term (bank) loans – "debt"
- Issuance (sale) of corporate notes or bonds
- Issuance (sale) of equity

One less recognized manner in which an asset can be acquired is by means of a *lease*, the discussion of which follows next.

For information regarding the calculation of the External Funds Needed requirement, please refer to the *Introduction to Financial Analysis* text by this author, available online on Pressbooks (https://pressbooks.pub/introductiontofinancialanalysis).

4.3 Growth and Assets Acquisition

We have noted that, theoretically, companies have an insatiable appetite for growth; by this we mean that companies wish to promote increases in sales and, presumably profits, ad infinitum. They are never fully satisfied and always want more.

In discussing the *External Funds Needed* model, we observed that, in order to grow, companies must increase their short- and long-term assets. More assets are required in order to produce more goods and services. This should lead to increased profits.

We shall identify three means by which long-term assets[1] may be acquired: with cash, with debt financing, and via leasing. Each of these choices affect *Capital Structure*. Naturally, the acquisition may also be done in some combination of these three, but we will not discuss that possibility here. For the moment, our interest resides solely in the effect that the undivided choice of the asset acquisition financing method has on the company's balance sheet.

In this sub-chapter, we will discuss the intricacies of *Leasing*, and then return to the more general topic of *Financial Leverage* using a combination of debt and equity. We will first note that leasing involves a capital structure element and decision.

Cash Acquisition

Should the company choose to purchase the asset with cash, the cash account will be reduced (credited) to pay for the asset, and the (long-term) assets account will be increased (debited) by an amount equal to the new asset's cost. The firm's total assets will be unchanged as will its debt ratios. If it decides to pay cash, it must figure into the decision the opportunity cost of cash. This assumes, of course, that the company has sufficient cash to pay for the asset's acquisition.

Cash	↓	Debt	No Change
Long- Term Assets	↑	Equity	No Change
Total Assets	No Change	Debt Plus Equity	No Change

Debt Financing

Alternatively, a company may borrow the money needed for the purchase – either from a lending institution or bank, or by means of a bond issuance. It may choose to borrow due to a lack of cash, or to preserve cash for necessary, or other even more productive, uses. In any event, the borrowing will result in both higher debt and assets levels. Leverage, that is to say, the amount of debt on the balance sheet, will be greater. The ability of the company to borrow once again in the future could be reduced to the extent it is already leveraged.

1. We will not concern ourselves here with short-term asset financing.

Cash	No Change	Debt	↑
Long- Term Assets	↑	Equity	No Change
Total Assets	↑	Debt Plus Equity	↑

To illustrate, let's say that XYZ Corp. decides to purchase a "fixed asset" for $5,000 and, rather than using equity (cash), it borrows via "long-term debt." What would the company's balance sheet (and debt ratio) look like before and after the asset purchase and debt acquisition? (For the debt ratio, use Total Debt (i.e., current plus long-term ÷ total assets.)

	Before	**Acquisition**		**Before**	**Acquisition**
Current Assets	$1,000	$1,000	Current Liabilities	$500	$500
Fixed Assets	9,000	14,000	Long-term Debt	4,500	9,500
			Equity	5,000	5,000
Total Assets	$10,000	$15,000	Total Debt + Equity	$10,000	$15,000
			Debt Ratio (D/TA)	5÷10=	10÷15=
				50%	67%

Notice that due to the asset purchase and debt financing, fixed assets, debt, total assets, and the debt ratio all go up. Take note also that the asset has been acquired with 100% debt and that no incremental equity was issued.

Leasing

Last, the company may choose to "acquire" the asset by leasing it. (It could also finance the asset with new equity, but we will not cover that alternative.) Leasing is not, in a legal sense, the same as a purchase, however it may be economically equivalent. An accountant, under current GAAP rules, must distinguish between an "operating" and a "financing" or "capital" lease. The accountant will not reflect any balance sheet entries if the lease is an operating one.

If a lease qualifies as a financing lease, the accountant will "capitalize" the leased asset. This involves reflecting some value for the asset on the balance sheet and recording a lease liability as well. For both the operating and financing leases, there will be both income statement (i.e., lease expense) and cash flow statement (i.e., lease payment) entries. Again, our focus here is only on the Balance Sheet and its leverage or capital structure effects.

Thus, leases may be used in lieu of debt, as a financing tool. A lease is not strictly debt in the sense that with debt the company borrows from either a bank or bond investors, and receives cash up front, which is then used for the asset acquisition.

However, in the case of a lease, the company (the "lessee") commits to a future series of payments for a stated time frame. Payments are made to the "lessor" who provides the company with the use of an asset. There is also often an asset service agreement. Thus, the lessee incurs a legal obligation, requiring that payments be made in the future, much like debt would require.

In the past, leases were considered an "off balance sheet" financing tool as there was no accounting recognition

of either the legal or its correlative financial events. Since the asset and its related debt were not recognized on the Balance Sheet, no negative impact was reflected in the company's debt ratios. As a result, the firm's financial status appeared stronger than it may actually have been. That policy changed in 1976.

4.4 Financing Leases

Omitting the asset from the balance sheet does not make the firm's total recorded assets any greater. Had the asset been recorded, this might have increased the company's borrowing capacity. By comparison, the omission keeps down the company's debt ratio. In other words, choosing to lease an asset has financial leverage implications.

The lease payments would still be reflected as expenses on the Income Statement and as Cash Outflow on the Cash Flow Statement. Outside analysts may thereby become aware of the lease and its financial implications, but there would be no explicit recognition of the lease apart from that.

In 1976, FASB 13[1] was passed in order to recognize that certain leases were both legally and economically speaking equivalent to long-term debt financing. In order for a lease to qualify as a "financing" or "capital" lease and, thereby, be recognized on the lessee's balance sheet, the lease must provide the lessee with at least one of the following benefits.

- Transfers ownership by end of term
- Includes a bargain purchase option
- The term is ≥75% of the asset's useful life
- PV of the lease payments ≥ 90% Fair Market Value of asset.

Take note that the lease payments constitute an annuity; hence, the present value of the lease payments are calculated as it was in the case of a mortgage. The payments would then consist of both principal and interest portions.

If a lease meets just one of the four foregoing criteria, it shall be treated, for accounting purposes, as a "Financing Lease." The underlying asset must then be "capitalized" by the lessee on the balance sheet, similar in manner to a debt-financed asset. If the lease does not meet any of the foregoing pre-requisites, it is deemed an "Operating Lease," and will not be reflected on the balance sheet at all. Operating leases are usually short-term.

TP[1]PT "FASB" stands for the Financial Accounting Standards Board," the rule-making authority for the accounting profession when this rule was promulgated.

In a financing lease, the lessee accounts for the asset as if it were acquired. That is, the asset is *capitalized* (rather than *expensed)* and *amortized*. "Capitalized" means that the asset will be carried on the Balance Sheet as a "Leased Asset." "Amortized" means that the asset's value will be reduced over time, in a manner similar to depreciation.

Capitalize

Amortize

The recorded asset value is equal to the present value of the committed lease payments. This is usually, but not always, approximately equal to the market value of the asset. The lessor records a "sale" of the asset at the inception of the lease.

A corresponding liability, "Lease Obligation," is created on the lessee's balance sheet. The amount equals that of

1. "FASB" stands for the Financial Accounting Standards Board," the rule-making authority for the accounting profession when this rule was promulgated.

the recorded asset. The discount rate used is that of the prevailing borrowing rate. Thus, the initial carrying value of both the asset and the liability are the same. Let us see, by way of example, how all this works.

4.5 Financing Lease Problem

You are given the following:

- LCM, Inc. is going to lease some equipment for 15 years at an *annual* charge of $200,000 – payable at the *end* of each year.
- There will be no salvage value
- LCM's borrowing rate is 5%.
- The lease satisfies all the criteria of a Capital Lease.
- The "Before" balance sheet is noted below.

To do:

1. Fill-in LCM's balance sheet and debt ratio at the lease's inception.
2. Question: What would the balance sheet look like one year into the lease (i.e., "year later")?
    - Assume straight-line depreciation for the asset.
    - Remember: the lease obligation is amortized at 5%.
    - Assume that the borrowing rate and lease rates are the same.
    - This is a two-part question.
        - One part has to do with calculating the depreciated asset value of the lease.
        - The other part has to do with the amortized lease obligation. They are not the same.
        - Therefore, an adjustment will need to be made to the balance sheet in order to make it balance.

(000)

	Before	**Inception**	**Year Later**		**Before**	**Inception**	**Year Later**
Current Assets	$200			Current Liabilities	$100		
Leased Equipment				Lease Obligation			
Fixed Assets	1,800			Long- term Debt	900		
				Equity	1,000		
Total Assets	$2,000			Total Debt + Equity	$2,000		
				Debt Ratio	50%		

4.6 Financing Lease (Solution to Question #1)

We already learned how to calculate payments for an amortized loan or mortgage. The relevant formula, you will recall, is:

Principal = Periodic Payment x Present Value Annuity Factor

Further, you will recall, that from the above formula, we derived the following:

Periodic Payment = Principal/PV Annuity Factor
Interest = Opening Balance x Rate

Principal Payment = Periodic Payment less Interest
Balance = Beginning Balance less Principal Payment

Using what we know about the Time Value of Money and the calculation of amortized loans, the balance sheet value of the lease is the PV of the annuity stream:

Annual Payment × PVAF (5%, 15 years) =
$200,000 × 10.3797 = $2,075,940

The balance sheet would reflect these figures, as noted below:

(000)

	Before	**Inception**		**Before**	**Inception**
Current Assets	$200	$200	Current Liabilities	$100	$100
Leased Equipment		2,076	Lease Obligation		2,076
Fixed Assets	1,800	1,800	Long- term Debt	900	900
			Equity	1,000	1,000
Total Assets	$2,000	$4,076	**Total Debt + Equity**	$2,000	$4,076
			Debt Ratio	50%	75.46%

4.7 Lease (Solution to Question #2)

First, take note that the solutions presented below will differ from the manner in which an accountant may record the lease's goings-on. We have ignored below the cash outflow emanating from the periodic lease payments. Similarly, we have ignored any cash inflows that will devolve from the productivity of the leased asset. We have done so in order to focus exclusively on the manner in which a lease will affect the firm's leverage.

This is a two-part question concerning both 1. depreciation and 2. amortization.

1. Depreciation of Leased Asset

The straight-line deprecation is: $2,076 ÷15 = $138.40

Therefore, the new asset value is: $2,076 – 138 = **$1,938**

Here are the relevant accounting book entries for the lease's deprecation – in the first year.

Income Statement / Depreciation Expense (dr)	$138.40
Balance Sheet / Accumulated Depreciation (cr)	$138.40

The balance sheet will reflect the following:

Gross leased equipment	$2,076
Accumulated Depreciation	(138)
Net Leased Equipment	$1,938

2. Amortization of Lease Obligation

In order to calculate the amortization of the lease obligation, we must figure out how much of the annual payment is principal and how much is interest. The interest is: .05 × $2,076 = $103.80

Total annual (cash) payments on the lease are **$200**. The income statement will reflect interest expense of $104 (rounded off), leaving $96 as amortization (reduction) of the principal. Therefore, the new lease obligation (liability) is: 2,076 – 96 = **1,980**. In accounting terms, we credit cash $200, and we debit both interest expense **$104** and amortization of the lease liability **$96**.

Another way of looking at these calculations is to utilize the approach we used to calculate mortgages. A lease is amortized just like a mortgage. This calculation is illustrated in the following table:

Year	PMT.	Interest	Amortization	Balance
	Credit to Cash	Debit to Income Statement	Debit to Balance Sheet	
0	–	–		$2,076
1	$200	(.05)($2,076)= **$104**	(200-204)= **$96**	($2,076-96)= **$1,980**

With this, one year into the lease, the balance sheet will look as follows.

(000)

	Inception	A Year Later		Inception	A Year Later
Current Assets	$200	$200	Current Liabilities	$100	$100
Leased Equipment	2,076	1,938	Lease Obligation	2,076	1,980
Fixed Assets	1,800	1,800	Long-term Debt	900	900
			Equity	1,000	958
Total Assets	$4,076	$3,938	Total Debt + Equity	$4,076	$3,938
			Debt Ratio	75.46%	75.67%

While the equipment and the obligation are depreciated and amortized at different rates, they will each sum out to zero at the horizon of the lease. In the first year of the lease, the "Leased Equipment" decreased by $138, whereas the Lease Obligation" decreased by $96 and the Equity decreased by $42 (i.e., 96 + 42 = 138).

The new equity may initially and incorrectly be thought of as a plug number, which is reconciled as per below. In this case, the plug number is a debit (accounting adjustment), representing the fact that the accumulated depreciation exceeds the lease obligation by that amount. At some point in the future, that relationship will reverse.

In fact, the firm's net assets, calculated as assets minus liabilities, will have decreased by $42. Here, the leased asset depreciated faster than the lease obligation was amortized. In this example, assets went down by $2,076 – $1,938 = $138, while liabilities decreased by $2,076 – $1,980 = or $96. This is a decrease in net assets of $138 – $96 = $42, as assets decreased more than liabilities. Therefore, it is reasonable to reduce the equity, in this case by $42, in order to match the net assets' reduction. If net assets decreased by $42, the equity must decrease by the same amount due to the application of this slightly modified basic accounting equation:

$$\Delta A - \Delta L = \Delta E$$

($138) – ($96) = ($42)

***Decrease* in Assets**	***Decrease* in Liabilities**	***Decrease* in Net Assets (A-L)**	***Decrease* in Equity**
$2,076 – $1,938= $138	$2,076-$1,980 = $96	$138- $96= **$42**	**$42**

This analysis may be expressed differently in terms of debits and credits as follows:

Reduction in leased asset – accumulated depreciation on balance sheet (credit)	138
Reduction in lease obligation- liability on balance sheet (debit)	(96)
Reduction in Equity – plug (debit)	(42)
Net	0

Summary Steps: Let's review the steps we took in order to resolve the Leasing Exercise.

1. Calculate the Present Value of the lease payments. Insert this PV into both the Leased Asset and Lease Obligation cells at Inception.

Using the mortgage formula:

Lease (PV) = (x) (PVAF)

PV = (200) (10.3797) = $2,076

2. Determine the Depreciation Expense. Subtract this expense form the Leased Asset's balance at Inception. Insert the new number into the Asset balance "One Year Later."

Straight-line Depreciation Expense: $2,076 ÷15 = $138

New Lease Balance: $2,076 – 138 = $1,938

3. Determine the Lease Obligation Expense by using the firm's opportunity borrowing cost. Subtract this amount from the annual Lease Payments in order to determine the Amortization in the first year. Subtract the Amortization from the Lease Obligation. This is your Lease Obligation One Year Later.

Interest = (.05) ($2,076) = $104

Amortization = Payment less Interest: $200 – 104 = $96

New Lease Liability Balance: 2,076 – 96 = $1,980

4. Adjust the Equity account. You can do this using the Basic Accounting Equation, or simply by plugging in a new Equity figure to force the Balance Sheet to balance.

Δ (A – L) = Δ E

($138) – ($96) = ($42) Equity is reduced!

4.8 Leasing Summary Calculations

The Leasing Problem, as conceived from a ceteris paribus perspective, involved just three steps.

Depreciation

The principal was calculated as $2,076. The assumed life is 15 years.

$2,076 ÷ 15 = $138 (Depreciation Expense)

$2,076 – 138 = $1,938 (New Asset Balance)

Amortization

The interest rate is 5%. The annual lease payment is $200.

($2,076) (0.05) = $104 (Interest)

$200 – $104 = $96 (Amortization)

$2,076 – 96 = $1,980 (New Liability Balance)

Adjustment to Equity

The basic accounting equation is: **A – L = E**.

Therefore: **ΔA – ΔL = ΔE**.

Assets decreased by $138, the amount of the depreciation. Liabilities decreased by $96, the amount of amortization.

(138) – (96) = (42)

$Eq_{Beg.}$ = $1,000

Less: (42)

Eq_{End} = $958

4.9 Examination of the Lease Obligation over its Entire Life

The table below illustrates how the Lease Obligation (Liability) presented above would look over its entire fifteen-year life. It functions like a mortgage because it has regular annuity payments which include both interest and amortization of the principal. Building further on this same example, we continue as follows:

	($000)			
Year	**Lease Payments**	**Interest @ 5%**	**Amortization**	**Balance**
0				2,076
1	200	104	96	1,980
2	200	99	101	1,879
3	200	94	106	1,773
4	200	89	111	1,662
5	200	83	117	1,545
6	200	77	123	1,422
7	200	71	129	1,293
8	200	65	135	1,158
9	200	58	142	1,016
10	200	51	149	867
11	200	43	157	710
12	200	36	164	546
13	200	27	173	373
14	200	19	181	192
15	200	10	192	0
Totals	3,000	924	**$2,076**	

As we know, the depreciation and amortization rates are unequal. As a result, the Balance Sheet will not balance since the assets and liabilities will be reduced by unequal dollar amounts. Therefore, we will need to adjust the equity account. This adjustment makes sense in terms of the basic accounting equation discussed above.

The Equity account will be adjusted each year, as noted in the table below. (Once again, please excuse rounding errors.) See the table below.

4.10 Examination of the Equity Account over the Term of the Lease

This table illustrates the annual calculation of the change in the Equity account and its correlative effect on the Equity account balance over the life of the lease. We have, of course, assumed ceteris paribus.

($000)

Year	Str. – Line Deprec. Exp.	Acc. Deprec.	Amort.	Acc. Amort.	Δ(A-L)=	ΔEquity	Equity Balance
0							**1,000**
1	138	138	96	96	(138)-(96)=	(42)	958
2	138	276	101	197	(138)-(101)=	(37)	921
3	138	414	106	303	(138)-(106)=	(32)	889
4	138	552	111	414	(138)-(111)=	(27)	862
5	138	690	117	531	(138)-(117)=	(21)	841
6	138	828	123	654	(138)-(123)=	(15)	826
7	138	966	129	783	(138)-(129)=	(9)	817
8	138	1,104	135	918	(138)-(135)=	(3)	814
9	138	1,242	142	1,060	(138)-(142)=	4	818
10	138	1,380	149	1,209	(138)-(149)=	11	829
11	138	1,518	157	1,366	(138)-(157)=	19	848
12	138	1,656	164	1,530	(138)-(164)=	26	874
13	138	1,794	173	1,703	(138)-(173)=	35	909
14	138	1,932	181	1,884	(138)-(181)=	43	952
15	138	**2,076**	192	**2,076**	(138)-(192)=	54	**1,000**

In summary, we note the following:

- Question: Ceteris paribus (assuming all else equal), will the equity in fact continually change as shown here?
    - Take note how the Equity decreases annually at first, and then increases after a time.
    - What effect will this phenomenon have on the corporation's Valuation?
- Note that, over fifteen years, the company's Equity has not changed – ceteris paribus! At the end, we are exactly back to where we started.
- Beyond the lease and its effects, there are many other things that are going on here, which are not noted.
    - In particular, the corporation took on the lease in order to increase sales and profits. Our default assumption has always been that the corporation has a never-ending appetite for growth. It chose to lease the asset rather than purchase it with borrowed money.
    - To the extent that the firm generates and retains profits from the leased asset, the Equity account will further change, hopefully in a positive direction.

In practice, we must evaluate whether it may be preferable to lease, or purchase and debt-finance the asset. Rarely, if ever, will the firm pay for the asset in cash due to the cost involved. The financial analyst would focus on whether the present value of a lease is greater or less than the present value of a purchase with debt financing. The alternative with the lower present value would be the cheaper and preferred alternative.

Other Considerations

One must consider that bank borrowing rates and leasing rates may not be equal. Actual residual values may differ depending on whether the firm purchased or leased the asset. An "owner" may take better care of the asset than a lessee, and project a greater residual value.

This raises some difficult qualitative questions.

- What is the asset's intended use? Will that subject the asset to an unacceptable level of wear and tear, including degradation?"
- Will the lessee abuse the asset because he does not own it?
- Is there a valuable service contract attached to the lease?
    - Leasing companies often provide a complete service contract to ensure the maximum ongoing working condition and value of the asset.
- Will the asset's productive life likely extend beyond the lease's term?
- Is this an asset, which is subject to technological improvement every few years, making ownership less attractive?

Our discussion has focused on an amortized lease. The scenario would look different for a simple, bullet loan or for a cash purchase. A bullet loan is one that requires interest-only payments, and no principal payments, over the life of the loan. At the loan's term, the entire principal must be paid. This makes annual payments, or cash flow requirements lighter, but leaves a great financial burden on the table later.

The asset may also be paid for in cash, in which case one must assess the opportunity cost of using cash. The opportunity cost represents the investment alternative forgone due to having used cash for the subject asset. New equity may also be issued to purchase the asset, in which case the relevant cost would be the external cost of equity capital.

Financing leases may be a means, especially in conglomerates, of transferring an asset to a higher tax bracket entity. This is usually a subsidiary. High effective tax rate firms tend to capitalize leases. The higher-bracket subsidiary receives a greater benefit from deducting (expensing) the lease payments from its income. At the same time, the lower-bracket subsidiary, which receives the income, is taxed at a lower rate.

4.11 Comparative Asset Financing Options Advantages and Disadvantages

Benefits / Demerits of Operating Lease

(A) A small – or no – cash outlay is required.

(B) Lessee may lease property for only a portion of its economic life; an operating lease can be useful for temporary equipment needs.

(C) Lessor retains risk of ownership; there may be a purchase option.

(D) Lessor's expert service is available.

(E) Lessee avoids capitalization and liability recognition, with, presumably the negative impact of the Lease Obligation on the firm's debt ratio. This may positively impact ROI, given lower recorded investment and higher potential inflows.

(F) There is less concern about violating any restrictive bond covenants.

(G) Lease payments are a tax-deductible expense.

Benefits / Demerits of Financing Lease

(H) Typically, higher long-term cost; no build-up of any equity.

(I) Lessee capitalizes and depreciates the asset.

(J) Interest costs are typically higher than on debt.

(K) Lease payments are recorded as "principal and interest" and may be a tax-deductible expense.

(L) If tenable, this serves as a means of transferring an asset to a higher tax bracket entity (depending on business opportunity and the state of tax laws).

- Historically, higher tax bracket entities tended to capitalize leases to take advantage of the tax-deductible depreciation expense. Long-term leases tend to be higher than short-term leases, thus the tax deduction is beneficial.

(M) Lessee, in many cases, cannot make improvements to leased property, as s/he is not the owner.

Benefits / Demerits of Buy and Borrow

(N) Lease Payments are fully tax-deductible as "lease expense".

(O) Only the interest portion of the loan, but not the amortization, is tax-deductible.

(P) The firm will depreciate an owned asset, most probably, at a different rate than it amortizes the leased asset.

(Q) The interest rate on a loan may differ from the internal lease rate (implied by its IRR)

4.12 Review Questions: Chapter Four

Review Questions: Chapter Four

1. In what manners are Operating Leases and Capital Leases accounted for differently?
2. In what three ways can a firm pay for a capital asset acquisition?
3. You are given the following Capital Lease information. Calculate the value of the capitalized asset and lease obligation on the Balance Sheet. Construct your own template.
    - HIZ, Inc. is going to lease some equipment for 3 years at an *annual* charge of $400,000 – payable at the *end* of each year.
    - There will be no salvage value
    - HIZ' borrowing rate is 5%.
4. Amortize the liability balance for each of the three years.

Solution to Questions #3 #4:

Year 0: PV = ($400,000) (2.7232) = $1,089,280 (Opening Liability Balance)

Year 1: (0.05) ($1,089,280) = $54,464 (Interest)

$400,000 (Payment) minus $54,464 = $345,536 (Amortization)

$1,089,280 – 345,536 = $743,744 (Liability Balance)

Year 2: (0.05) ($743,744) = $37,187 (Interest)

$400,000 – 37,187 = $362,813 (Amortization)

$743,744 – $362,813 = $380,931 (Liability Balance)

Year 3: (0.05) ($380,931) = $19,047 (Interest)

$400,000 – $19,047 = $380,953 (Amortization)

$380,931 – $380,953 ≈ 0 (Final Liability Balance – with small rounding error)

Chapter 5: Financial Leverage

5.1 Chapter Five Learning Outcomes

Learning Objectives

- **Identify** how the *Crossover Point* affects decision-making.
- **Explore** the interaction of Leverage and Risk Measures.
- **Consider** the variables that go into a firm's *Optimal Capital Structure*.
- **Connect** the firm's Capital Structure with its Valuation.
- **Calculate** the effects of Leverage on Net Income, Return on Equity, and Earnings per Share.
- **Observe** the effect of increased leverage on corporate risk.
- **Identify** the Optimal Degree of Leverage for a company's marginal project investments.

5.2 Financial Leverage

So far, we have discussed the company's WACC. Let us examine from a managerial perspective, the pros and cons of increasing the company's leverage, or use of debt as a financing source of growth (think again: "EFN"). We shall see, by way of example, that leverage affects net income, EPS and ROE.

As a point of departure, we noted earlier that debt is the cheapest capital component. If so, why not leverage (i.e., increase debt) the company "all the way, i.e., to 100%"? We will see below that leverage increases interest expense, which decreases net income, a negative. However, under certain circumstances, which we shall see, leverage also increases EPS and ROE, positives, which should favorably affect share price, which are driven by earnings and dividends (assuming a constant payout ratio), as reflected in the DDM and the PE ratio.

We are given two alternate cases (Cases 1 and 2) for different capital structures – no debt, and 50/50 debt to equity. We are also given varying *projected* EBIT levels A, B, and C as below.

Case 1: 0% Debt, 100% Equity (10,000s. @ $20 book value per share)

Tax Rate ("T") = 0.40

Case 2: 50% Debt, 50% Equity ($100,000 of debt; 5,000s. @ $20)

i = 0.12 and T = 0.40

($000)

Pro-forma Income Statement	Case 1			Case 2		
	A	**B**	**C**	**A**	**B**	**C**
EBIT	0	40	80	0	40	80
Int Exp	0	0	0	(12)	(12)	(12)
T	0	(16)	(32)	4.8	(11.2)	(27.2)
NI	**0**	**24**	**48**	**(7.2)**	**16.8**	**40.8**
EPS	**0**	**$2.40**	**$4.80**	**($1.44)**	**$3.36**	**$8.16**
ROE	**0**	**12%**	**24%**	**(7.2%)**	**16.8%**	**40.8%**

Note:

EPS= NI ÷ NOSO

ROE= NI ÷ Equity

NOSO= the number of shares outstanding

You will note that, in Case 2A, taxes are stated as positive "4.8." In effect, this means that the company will earn a tax "carry back" (similar to a credit on past taxes paid) or a tax "carry forward" against future tax liabilities. Taxes are usually paid; here, the corporation gets a tax benefit as either a refund on past taxes paid or a decrease in future taxes to be paid.

Let's compare the results case by case. The chart below provides a summary.

	EBIT	**Net Income**	**EPS**	**ROE**
2A vs. 1A	**$0**	**Worse**	**Worse**	**Worse**
2B vs. 1B	**$40,000**	**Worse**	**Better**	**Better**
2C vs. 1C	**$80,000**	**Worse**	**Better**	**Better**

In all instances, the leveraged case (Case 2) increases interest expense (here from zero to $12,000), and thereby reduces net income. Case 2 is "worse" across the board. This says that, in terms of net income, the company is worse off with debt.

However, somewhere between EBIT of zero and $40,000, leverage enhances both EPS and ROE. In other words, leverage is desirable if you expect pro-forma EBIT to be beyond a certain level, a level, which we will determine mathematically below. In short, the negative impact of leverage on net income is overcome, in terms of EPS and ROE, by the relatively smaller number of shares outstanding in the presence of leverage. By using "other people's money" (i.e., debt) the shareholder gets to "keep," in a sense, relatively more earnings for himself. In accounting and financial language, the shareholders' equity interests are not "diluted" by the issuance of additional shares.

We note in summary:

- Debt (leverage) does not affect "operating risk" (i.e., EBIT and ROA variability).
- Debt increases "financial risk," and causes investors to demand higher ROE.
- Debt is a negative in that it reduces net profits, erodes the TIE ratio, and increases default and bankruptcy risks. (This assumes that the component cost of debt is not increased with more debt.)
- Debt is cheaper than equity; increased debt will lower a firm's weighted average cost of capital, or "WACC."
- Debt is a positive, in that it enhances EPS and ROE – *provided that* the EBIT exceeds the "crossover point" (i.e., in this example somewhere between $0 and $40,000). If EBIT does not exceed the crossover point, debt is ill-advised.
- Debt does not "work" unless it adds to operating profits in a manner, which is greater than the cost of debt. When it does add profits, the benefits increase as operating profits increase.
- Debt does not "dilute" ownership interests. (We note, in passing that increased debt magnifies default risk.)

This framework may be used in corporate investment planning, i.e., the conceptual means by which an accepted capital project may be financed. We use the concept of "incrementalism" here to evaluate how much additional capital is needed and whether the requirement should involve debt or equity (or some combination). Thus, the $200,000 capital figure alluded to in this example, may be incremental and for the sole purpose of financing a particular capital project.

Note:

Some Important Financial Leverage Ratios:

In the *Introduction to Financial Analysis* text (by this author), we reviewed numerous financial ratios and, in particular, we looked at three Debt or "Solvency" Ratios.

The solvency ratios are intended to provide a gauge of the extent to which the firm is indebted. Thus, we examine both the Debt/Equity and the Debt/Total Assets ratios. A higher solvency ratio means that the firm has a higher default risk than a lower ratio. "Default" refers to the failure of a company to pay its debt obligations in full and on time.

We also examine the "TIE" or "Times Interest Earned" ratio in order to gauge the firm's ability to generate sufficient *operating cash flow*, or EBIT, with which to pay its interest expense. The firm should have more EBIT than interest expense. EBIT ÷ Interest Expense should therefore exceed one.

EBIT and Operating profits are accounting data that are subject to the various accounting schemes which the auditor may legitimately employ under GAAP accounting. (Much space was devoted to these "schemes" in the above cited text.) Thus, we now introduce a third ratio: EBITDA ÷/Interest Payable. This ratio provides a more realistic view of the cash demands interest payments make and the firm's ability to honor those demands. This ratio should also exceed one.

EBITDA stands for earnings before interest, taxes, depreciation, and amortization. Taxes are paid after interest has been paid. Interest is a tax deduction.

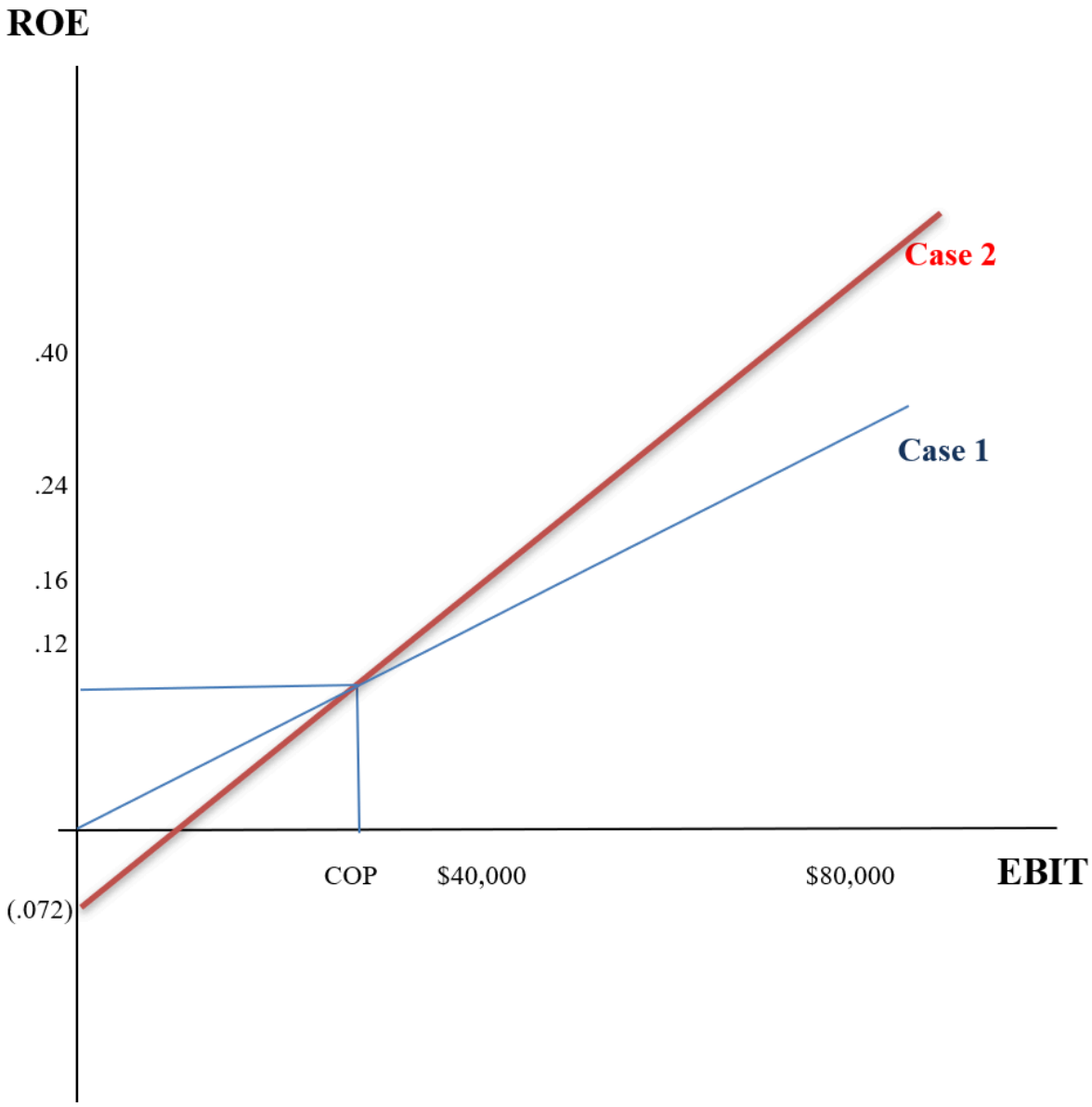
Depreciation and amortization are non-cash expenses that the accountant reports. In adding back these deductions, we get a truer image of the firm's operating cash flow ("EBITDA").

Summary Debt Ratios

Debt ÷ Equity	TIE = EBIT ÷ interest expense
Debt ÷ Total Assets	TIE = EBITDA ÷ interest payable

5.3 Financial Leverage (Graph)

Using the data for EBIT, ROE, and EPS on the prior pages, graph below the two cases provided. (You may plot EPS and ROE congruently with one another.) You will observe a "crossover point," i.e., a point at which both cases produce the same EBIT, on the horizontal axis. Note that when the projected EBIT exceeds the crossover point, the company's projected EPS and ROE are more favorable if leverage is employed – and vice versa. In the graph below, plot the data from two pages back. The EPS and ROE points will be congruent. (On the vertical axis, we have noted ROE values.) Note that the leveraged plan has a steeper slope.



The "crossover point" ("COP") on the horizontal axis is the point at which the two (or more) alternative plans intersect – as diagrammed above. To the right of this point, increased leverage is advised, as it will provide higher EPS and ROE. (At the crossover point, the ROE and EPS for each alternative respectively, will be the same.) That's what shareholders want! On the next page, we shall calculate the precise value of the crossover point.

5.4 The Crossover Point

The EBIT "crossover" point represents the unique level of EBIT for both plans, at which the EPS and ROE respectively will simultaneously be the same for both plans. In other words, at a certain EBIT ("the crossover point"), the EPS for both plans will be the same, as will the ROE be the same for each. (In fact, the crossover point will be the same regardless of the amount of leverage employed; we will see this later.)

We shall – arbitrarily – choose to use the ROE (rather than the EPS) formula in order to calculate the EBIT crossover point. (We know this choice does not matter as the choices – EPS and ROE – are congruent diagrammatically and mathematically identical; this too we will see later.) Using the prior example, we will set the unknown EBIT to equate to the unique ROE of each alternative, as follows, and then solve for the EBIT. In other words, we will set the ROE formula for each plan to be equal to one another and then solve for EBIT!

$$\mathbf{ROE_{Case\ 1} = ROE_{Case\ 2}}$$

$$\mathbf{ROE = NI \div Eq. = [(EBIT - Interest\ Exp.)\ (1- T)] \div Eq.}$$

Again, by definition, the EBIT value where the ROE (or EPS) of each alternative is the same *is* the crossover point! The formula for ROE, in both cases, is simple accounting (as above in general and as solved below):

ROE_1 = (EBIT – 0) (1 – .40) ÷ $200,000 =

ROE_2 = (EBIT – 12,000) (1 – .40) ÷ $100,000 = EBIT (.6) ÷ $200,000 = (EBIT – $12,000) (.6) ÷ $100,000

To simplify, let's solve for EBIT:

$$(.6)\ (EBIT) / 200 = [(.6)\ (EBIT - 12)] / 100$$

$$(.6)\ (EBIT) / [(.6)\ (EBIT - 12)] = 2$$

$$EBIT = 2\ (EBIT - 12)$$

$$EBIT - 2\ (EBIT) = -24$$

$$EBIT = 24$$

When EBIT = $24,000:

$$ROE = NI/Equity$$

$$ROE_1 = 14.4/200 = 7.2\%$$

$$ROE_2 = 7.2/100 = 7.2\%$$

Here is a tabular summary:

	No Leverage	**50/50 Leverage**
EBIT	24	24
Interest Exp.	00	(12)
Taxes	(9.6)	(4.8)
Net Income	14.4	7.2
ROE	14.4/200 = **.072**	7.2/100 = **.072**
EPS	14.4/10 = **$1.44**	7.2/5 = **$1.44**

Note: EPS = NI NOSO

The exact value of the crossover point matters in terms of capital planning. If the firm projects an EBIT level to the right – or left – of the crossover point, the capital funding decision will be affected accordingly. To the right of the point, leverage is favorable.

We note that we have used just two cases above (i.e., no-leverage, and leverage set at 50%). In fact, we could use multiple cases, addressing increasing levels of leverage and risk. We could still calculate a crossover point – and it would be the same locus point regardless of the degree of leverage (see below)!

Finally, we ask that if debt under certain conditions maximizes EPS and ROE, how much debt should the company use? Although we have only illustrated 50/50 leverage, it is an easy step to show that more debt provides even greater earnings per share *leverage* – at greater risk!

Note 1:

Had we used, say, 25% leverage, the crossover point would still be the same. See if you can explain this solution and how it may relate to the choice of degree of leverage. (The solution below compares 0% and 25% leverage, but would work for, and come out with the same solution as, any other leverage ratio.)

$$[(EBIT)\ (0.6) \div 200] = [(EBIT - 6)\ (0.6) \div 150]$$

$$EBIT = \$24$$

Note 2:

We could have solved for the crossover point using EPS rather than ROE. Here we go.

Again, let's assume that $EPS_1 = EPS_2$.

- And **EPS = [(EBIT – I) (1 – T)] ÷ NOSO**
    - ("NOSO" = number of shares outstanding)
    - Only the denominator is different in this formulation
- This time, we'll substitute "x" for EBIT. Next….
- [(x – 0) (1 – 0.40)] ÷ 10,000 = [(x – 12) (1 – 0.40)] ÷ 5,000
- Skipping along….
- X = 2 (x -12)
- X = EBIT = 24

5.5 Leverage and the Crossover Point

We observed above that the crossover point – on the horizontal axis – corresponds with certain values in terms of EBIT, ROE, and EPS – on the vertical axis. This point was noted for the comparison of no leverage, 50/50, and 25/75 (see footnote above) leverage cases. In fact, using these cases, we observed that any degree of leverage yielded the same crossover point! In other words, there is a certain proportionality, which pertains to this, and any, example.

To illustrate this point further, let's imagine that the firm considers using 80% leverage ("80/20") for this project. You will find that the vertical intercept is yet lower, that the line passes through the former crossover point, and that, overall, the line is steeper than the 50/50 leverage line. This is as it should be, because 80/20 leverage is riskier! When EBIT is "great," i.e., to the right of the crossover point, EPS and ROE are more and more favorable the more leverage there is, but beware, this is also true in the reverse!

Let's work-up the numbers using $24,000 as our EBIT, as below. Next, let's insert the new, 80% line in the diagram above. (Do this on your own, as before, using various EBIT values of, say, $0, $40, and $80, and then calculating EPS and ROE. Once you have done this, insert the values into the graph on the prior pages.)

	Degree of Financial Leverage			
($000)	**D/E= 0/100**	**25/75**	**50/50**	**80/20**
EBIT	**24**	**24**	**24**	**$24**
Int. Exp.	00	(6)	(12)	(19.2)
Taxes	(9.6)	(7.2)	(4.8)	(1.92)
Net Inc.	14.4	10.8	7.2	2.88
ROE	14.4/200=**.072**	**.072**	7.2/100=**.072**	**.072**
EPS	14.4/10=**$1.44**	**$1.44**	7.2/5=**$1.44**	**$1.44**

Note: ROE and EPS are the same regardless of the degree of leverage!

Worksheet (for the above table): Assume that EBIT = $24,000

	80/20 Leverage	**25/75 Leverage**
Interest Expense	(0.12)($200,000)(0.80)= $19,200	(0.12)($200,000)(0.25) = $6,000
Taxes	($24,000 -19,200)(0.40)= $1,920	($24,000 – 6,000)(0.40) = $7,200
ROE	($2,880) ÷ ($200,000)(0.20)= 0.072	($10,800) ÷ ($200,000)(0.75) = 0.072
EPS	($2,880) ÷ (10,000s.)(0.20) = $1.44	($10,800) ÷ (10,000s.)(0.75) = $1.44

Notes:

- We have made a critical, “simplifying,” ceteris paribus assumption, namely that the firm’s interest rate does not go up with increased leverage. This may – or may not – be true!
- EPS = (NI – Preferred Dividends) ÷ “NOSO”

5.6 Leverage and Risk

Commonly, we think of risk as the possibility of losing money. While there is a great deal of sense in that assertion, we look at risk differently in traditional Finance.

In Finance, "risk" is defined as volatility of earnings or return, by which here it is meant EPS and ROE. Volatility! Volatility is two-directional. If risk – leverage – is great, then EPS and ROE will vary greatly – up or down – for a given change in EBIT. This movement corresponds to the slope of the various leverage lines. (A common, but not exclusive, measure of risk is the standard deviation of the variable.)

Let's look at some numbers; fill in the blank cells. This way, we will be able to map out our entire set of Leverage lines. We will focus on ROE as before; EPS and ROE will be congruent as on the prior graph – on the vertical axis. An ROE *Sample Solution* is provided on the page below.

ROE Table

With Varying Levels of EBIT and Leverage

		Degree of Financial Leverage			
		0/100	**25/75**	**50/50**	**80/20**
EBIT	**0**	0%		(7.2%)	
	24	7.2%	7.2%	7.2%	7.2%
	40	12%		16.8%	
	80	24%		40.8%	

Notice how to the right of the crossover point, i.e., where EBIT = $24,000, leverage increases ROE *increasingly*, i.e., the vertical distance between the different plans' lines widens! If you graph this on the prior page, you will note that all ROE lines pass through the crossover point, and that higher leveraged lines are steeper, representing greater risk.

In the foregoing case of financial leverage, the steeper the EPS/ROE line the greater the risk because there will be more movement along the vertical axis for any given change in the horizontal axis. 80/20 leverage is steeper than 50/50, which is steeper than 0/100. (The standard deviation of both ROE and EPS will be greater the steeper the line.) Again, and in other words, for any change in EBIT – left or right – we will have *more* volatility in EPS and ROE – more up or down. *That* is volatility! *That* is, what we, finance people, mean by "risk"!!

Slope	**Financial Risk**	**Where slope = Δy/Δx** **=Δ ROE (or Δ EPS) / Δ EBIT**
Shallow	Low	
Steep	High	

In order to fill in the table, you must set ROE as the outcome variable (the dependent variable) to the right of the equal sign in the ROE formula. We know that ROE = NI / Equity = [(EBIT – *i*) (1 – T)] / Equity. Interest expense

will change as will the amount of Equity, but not the Tax Bracket (T), with changes in the Degree of Leverage. On the next page, we provide a sample solution for one cell in this table.

ROE Table

(Solutions)

		Degree of Financial Leverage (DOL)			
		0/100	**25/75**	**50/50**	**80/20**
EBIT	**0**	0%	(2.4%)	(7.2%)	(28.8%)
	24	7.2%	7.2%	7.2%	7.2%
	40	12%	13.6%	16.8%	31.2%
	80	24%	29.6%	40.8%	91.2%

With increased leverage, there is more movement up or down the vertical axis for every one-dollar decrement/ increment in EBIT – along the horizontal. This means that you may potentially earn more EPS and ROE as EBIT increases, but also risk a greater negative impact should EBIT come in lower. Yes, it is true that as management projects EBIT to exceed the crossover point, increasing leverage may be warranted. Still, the degree of leverage actually chosen is a decision function of management based on its own (and shareholders') risk profile(s), as well industry characteristics and relative capital costs.

Plot your solutions on a new graph: EBIT on the horizontal axis and ROE on the vertical. You now have four lines, whereas earlier we had only two DOL cases and two lines (for 0/100 and 50/50 leverage), and we did not then know the precise value for EBIT at the Crossover Point. Again, the EPS and ROE points will be congruent if you plot all the lines. The 80/20 DOL line is the steepest because it is the riskiest. With more leverage, you potentially get more EPS and ROE – if EBIT works out as predicted. Do you have the stomach to bear the negative effects if projections fall far short?

With a steeper, more risky line, you get more movement along the vertical axis for every change along the horizontal than with a less steep line. If we get a change along the horizontal, we will get more of an increase in the vertical than with a shallower line. However, take note that it works in both directions. A steeper line will also get more downward movement. This makes the steeper line riskier. The 80/20 mix is riskiest and the numbers also bear that out!

Were we instead solving for an "EPS Table" – rather than the "ROE Table" as above, we would use a different formula, but not much different. Here it is: EPS = NI / NOSO (where "NOSO" is the number of shares outstanding). Once again NI = [(EBIT – i) (1 – T)].

Question: What if either interest rates and/or taxes increased or decreased?

1. If taxes went up, the ROE would go down.
2. If rates went up, Net Income would go down.

5.7 ROE Sample Solution

Here is the solution for: 25/75, when EBIT = $40.

- We will set ROE as the outcome variable.
- ROE = NI / Equity = [(EBIT – *i*) (1 – T)] / Equity.
    - Since Debt is 25%, Equity is $150.
    - NOSO = (0.75) (10,000s.) = 7.5s.
- As Equity varies, so too will Interest Expense.
- The Tax Bracket (T), does not change, ceteris paribus, with variations in the Degree of Leverage (DOL) and Equity. Taxes remain 40%.

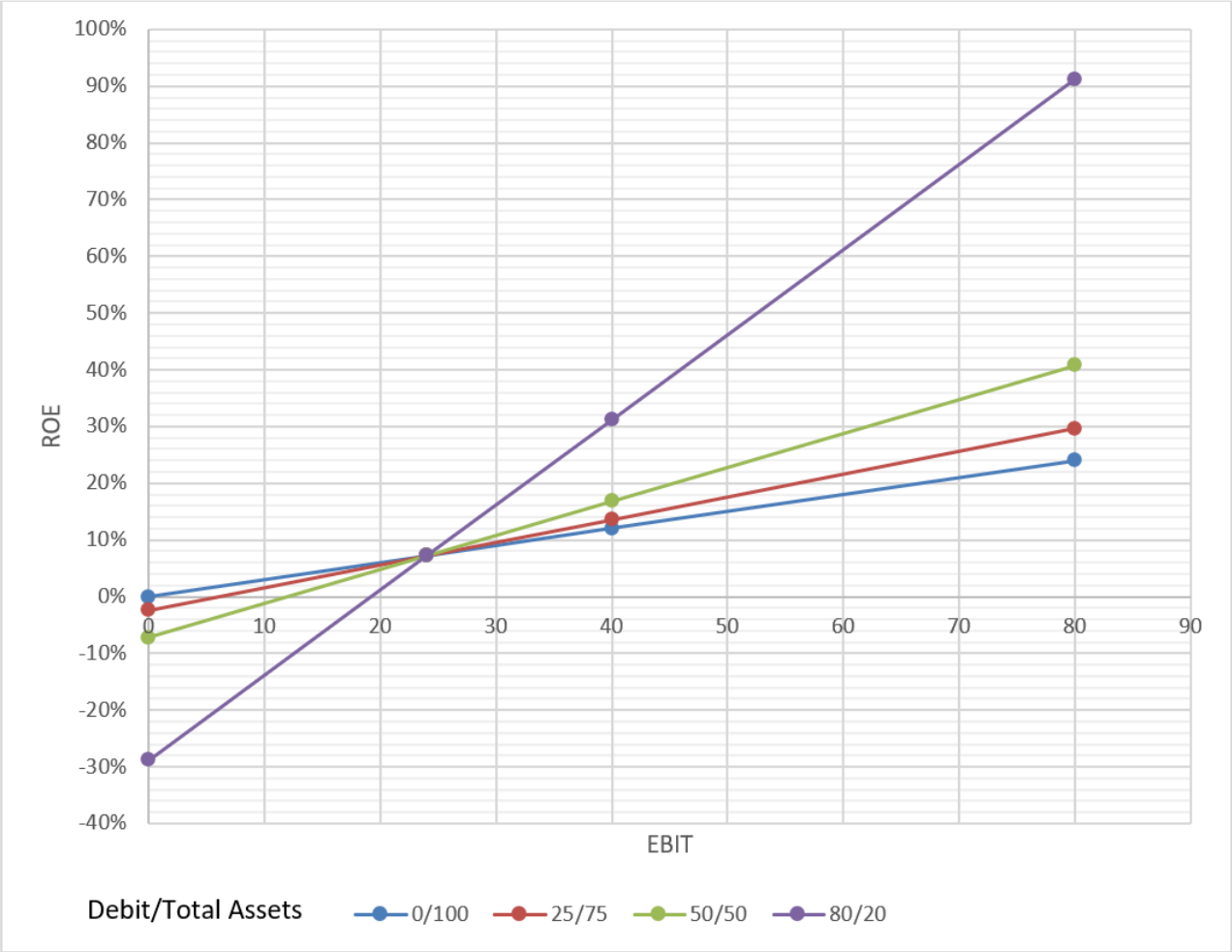
Once again, the calculation steps are:

		Calculations	**Formulae**
EBIT	$40	Assumed	
(Int)	(6)	(0.12)(.25)($200)	(Int. Rate)(% Debt)(Total Capital)
EBT	34	40-6	EBIT – Int. = EBT
(T)	13.6	(34)(0.40)	(EBT)(T)
NI	20.4	34 – 13.6	EBT – T

ROE	0.136	20.4 ÷ 150	NI ÷ Eq
EPS	$2.72	20.4 ÷ 7.5	NI ÷ NOSO

5.8 EBIT and ROE for Different Degrees of Leverage: A Diagram

The diagram below depicts the relationships between various levels of EBIT and its corelative ROEs given differing degrees of Leverage as calculated in the preceding pages. Take note that all DOLs pass through the same crossover point. If EBIT is forecasted to be significantly higher (to the right) of the crossover point, an aggressive management would choose a high degree of leverage (80/20) for that capital project.



- Many thanks to Shaya Gross and Yitzy Lisker (LCM ('22) for providing this diagram.

5.9 Implications of Financial Leverage

Leverage, or the increased use of debt as a financing source, reduces net income because of the added interest expense debt entails; the effect on net income is mitigated, but not eliminated, due to the tax deductibility of interest expense. Further, debt increases financial risk as interest must be paid timely; this puts some stress on the company's solvency ratios. There must be a good reason to use debt. We have observed that the use of debt, in lieu of equity, potentially increases EPS and ROE. Shareholders should like that.

The guiding principle of the "optimal" or ideal level of leverage is to maximize firm (stock) value, i.e., minimize WACC (due to inverse price/discount rate relationship). Since the cheapest source of funds is debt capital, increasing debt would be best. It would also provide leverage for the shareholders, providing higher EPS and ROE, and share price – in theory.

However, as leverage increases so too does the risk of bankruptcy (as measured by the TIE and other ratios), which would, contrarily, diminish the value of the company's stock. This will cause the firm's equity Beta to rise, with the result that the "Required Return" ("R") from the CAPM for the firm will also go up. This "R," in turn, serves as the firm's discount rate in the dividend discount model. If the discount rate rises, the firm's intrinsic value (price) will decline. Neither too much – nor too little – debt is desirable.

Of course, there is more to this. As we have seen, leverage can be a good thing – providing more with less, so to speak. If the capital funds that debt provides are profitably utilized by careful capital budgeting and planning, the positive effect is that ROE and EPS will rise. If earnings (per share) rise, ceteris paribus (i.e., assuming a constant payout ratio), dividends will also rise. If dividends (and if the dividend growth rate – G) rise, price should also rise, via the Dividend Discount Model! So, as you see, there are offsetting considerations here.

Variability of EPS and ROE is greater in leveraged case, i.e., leverage magnifies gains and losses. (Note: Variability can be measured by standard deviation.) A leveraged company is riskier! Take note that the more leverage, the steeper the EPS/ROE versus EBIT line (see graph above); this means that EPS and ROE move more for every unit change in EBIT than were the line flatter – the very definition of volatility!

This is a static analysis because it ignores the possibility that the cost of debt-capital itself may rise as leverage increases (or as market-conditions / yield-curve may change).

If a corporation's capital structure does not match the structure an *investor* targets for himself, he can, as we shall soon see, create the amount of leverage he desires by either borrowing or lending with his *personal portfolio* and thereby retaining the personal payoff effects desired. This is sometimes called "Homemade Leverage."

According to Modigliani and Miller (M&M), capital structure hence will not affect stock price, in stark contrast with the above notion of leverage and its purported effect on price. "The size of the pie (the Balance Sheet) doesn't depend on how it's sliced." You will soon note M&M's counter argument (below) that the firm's riskiness is a function of the volatility of its basic (business) earnings power, EBIT (or equivalent).

5.10 Optimal Capital Structure

The “optimal” capital structure is the one that minimizes the weighted average cost of capital, or WACC. Capital Structure refers to the mix of debt and equity. Financial risk refers to the chance that the firm may become insolvent or go bankrupt.

What is the optimal capital structure? If a company has no debt, there would be no financial risk – only business, or operating, risk. (The company may still go bankrupt due to business causes, but not due to “financial” causes.) So, let’s use 100% Equity!

We also know that debt is the cheapest capital component. This fact would argue for using virtually 100% debt in the corporation’s capital structure, except for a minimal amount of equity to be had for the owners.

Once again, we recall that, as debt increases, so too does the risk of insolvency – and bankruptcy. As this risk increases, so too does the cost of debt in the manner of higher interest rates. Thus, the debt ratio should be less than 100% of total assets. There should be some debt because it is cheapest up to a point, but not too much! How much less? “It depends”! In summary, the ideal debt ratio is somewhere between 0-100%!

The horizontal axis ranges from 0% to virtually 100% debt as a percent of total assets. If the company has virtually

100% equity, the curve will intersect the vertical axis at some relatively high point on it. Draw in that point – on the axis.

Contrarily, if it had, say, 99.9% debt, it would have a coordinate high vertically and to the extreme right horizontally. As we continually decrease Equity – moving to the right, a decreasing curve emerges, i.e., curving downward to the right in a partial "U" shape. The increased use of debt will lower the WACC – up until a low point, at which point increased debt will subsequently come with a higher interest rate. As debt increases further, lenders perceive greater default risk and demand higher interest rates in order to compensate. The WACC will start rising. The curve too will start rising as it moves further to the right.

Can you draw in this optimization relationship in the graph above? It should appear as a "U," with the low point (i.e., the "optimization point") being the "optimal capital structure"; this is the lowest WACC. Another point will touch high up on the vertical axis.

To reiterate, as we increase the amount of debt from zero on the vertical axis to the right, the WACC will be reduced in some manner until it reaches its lowest point. Then, at its low point, as we continue increasing debt, and as the debt ratio increases, the WACC will increase in some proportion to the increase in the debt ratio – due to the increased default and bankruptcy risks, and the consequential rise in lending rates or market yields.

The effective use of leverage, i.e., the carefully measured use of debt, can increase the firm's ROE above its ROA (see the DuPont Model in your *Introduction to Financial Analysis* text by this author); with no debt, ROA = ROE. Remember that in the DuPont Model, ROA = NI / TA and ROE = NI / Eq. Net Income is in both the ROA and ROE numerators. Borrowing does not assure in all instances that ROE > ROA. This will occur if EBIT is projected to exceed the crossover point. It will also occur if the increase in EBIT exceeds the increase in Interest Expense.

Again, what is the optimal point, the ideal amount of leverage? We said earlier that "it depends." Unfortunately, there is no mathematical formula that may be implemented to solve for the low point on the "U"-shaped leverage curve. In fact, the actual, current, or existing capital structure is (not completely seriously) said to be the ideal one – otherwise the corporation's – rational – managers would not have chosen it! (oh, c'mon; really?) The choice of capital structure, in fact, depends on numerous variables, some of which are qualitative and immeasurable, as noted below.

Industry Characteristics	*Management risk profile*
Shareholders' risk profile	*Relative capital costs*

These characteristics will be discussed in proceeding sections. First, let us examine the impact of varying capital structures on "shareholder value" – share price. It is not so simple or straightforward. We will then discuss the four characteristics noted immediately above.

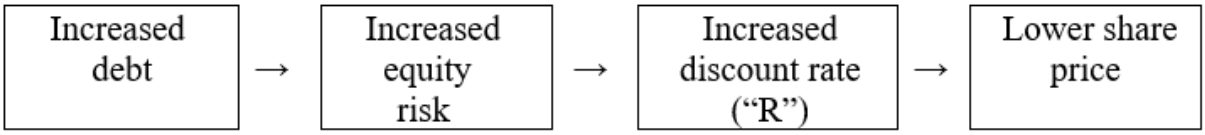
5.11 The Impact of Financial Leverage on Valuation or Price

The impact of financial leverage on a firm's valuation or stock price is, in fact, unclear.

On the one hand, if leverage increases, a firm's default risk may also increase (but then again maybe not!). This increased risk would make the company's equity riskier as well because the shareholders get paid only after the creditors do. There can be no dividends if interest on bonds is not paid; there can be no internal growth in investment and thus no capital gains if interest is not paid.

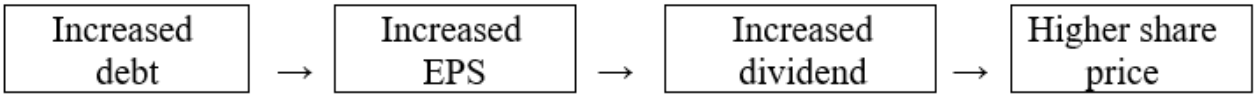
In turn, the firm's *required return*, i.e., the dependent "R" variable in the CAPM, would also rise since the CAPM formula embodies equity risk (β). (The firm's slope, as represented in the CAPM diagram, would steepen.) Finally, with a higher "R," which is also the discount rate in the DDM, the stock price would go down.

The table below summarizes the steps emanating from increased leverage, according to this argument. The relationships isolate the effect of debt on share price only, ceteris paribus.



Alternatively, increased leverage may provide greater EPS – when new debt-sourced funds are employed profitably, when the company institutes good capital budgeting decisions, or when EBIT is to the right of the crossover point. That is in fact, the reason for employing leverage – to increase EPS and ROE! Assuming a constant payout ratio, a ceteris paribus condition, the dividend would also increase and, via the DDM, the price will rise. In fact, both scenarios occur simultaneously and in varying degrees; the results depend on how well the firm employs its invested capital.

The table below summarizes the steps emanating from increased leverage, according to this argument. It is assumed that the borrowed funds are invested well so that EPS increases.



Whether we have one or the other outcome depends on how profitably the firm invests its (external and marginal) debt, which is a function of the efficiency of its capital budgeting process.

So, once again, does capital structure matter? It seems that it all depends – on the soundness of your Capital Budgeting decisions and on the volatility of EBIT!

5.12 Truth, Opinion, and ….

Photo by Ivan Rohovchenko on Unsplash

Everyone is entitled to his own opinion,
but not to his own facts.

-Senator Daniel Patrick Moynihan

5.13 The Importance of Capital Structure in the Firm's Valuation

We have demonstrated that leverage affects EPS and ROE; it should not be a tall order to connect these data with the firm's valuation via the dividend discount model or some alternate rubric. However, we have not been able to answer (universally) what the optimal firm leverage ought to be. We know that too much leverage increases default risk; that is not good. We also know that too little leverage does not offer favorable results in terms of increases in EPS and ROE – and share price!

We also know that if EBIT is expected to come in to the right of the crossover point that some degree is leverage is warranted. That still does not inform us how much leverage we should take. It turns out, once again, that the degree of leverage a firm chooses depends on four qualitative factors.

Much will depend on industry characteristics, including the stability of the industry's operating cash flows, which go to paying debt. For example, electric utility companies have captive and regular customers, and hence very stable operating earnings; these companies can tolerate high debt levels, which they need to finance costly power plants and thus will manifest "thin" TIE ratios.

Still more depends on management's risk profile. Can management tolerate aggressive solvency ratios in the pursuit of higher earnings? What about shareholders? If the company's leverage is too aggressive, or not aggressive enough, will shareholders sell, and subsequently buy shares in companies more suited to their respective personal risk profiles?

We may also ask whether leverage indeed matters at all? Must capital structure match the investor's personal risk profile and, if not, does capital structure really matter at all to a firm's worth? (Initially, we may think that leverage matters, because it affects the firm's WACC, its bonds' respective yields, its equity capital costs, and thus, via the numerous valuation models the firm's very worth!)

It can be shown, however, that an individual himself can emulate, in his investment portfolio, the capital structure of a company in which s/he is invested, by borrowing (or *lending*) money within his personal portfolio. For example, an individual may purchase stock in an un-leveraged company (i.e., a company with no debt, or 100% equity) and leverage his investment by buying the stock on margin. "Buying on Margin" means paying for as little as half the cost and borrowing the rest from one's stockbroker.

After accounting for the cost of borrowing (i.e., interest), but ignoring taxes, it will be shown that the earnings to the investor are the same. It will also be assumed that the investor's marginal borrowing rate is the same as the corporation's.) If so, arguably, leverage may not matter; the investor can create his/her own leverage, the result of which is the same EPS and ROE, and hence value (or valuation), to the shareholder! We'll see all this soon!

5.14 Review Questions: Chapter Five

Review Questions: Chapter Five

1. What effect does Financial Leverage have on Net Income, Earnings per Share, and Return on Equity? Explain.
2. Financial Leverage modifies the relationships between _____ and _____.
3. When observing the slope of the EBIT/ROE (or the EBIT/EPS) line, a steeper slope represents *lesser / greater* leverage. (Remember: EBIT is on the horizontal axis.) Which is it? Why?
4. Under what circumstance is leverage advised or ill-advised?
5. Calculate the Crossover Point, given the following:
    - Total Capital $2.5 million
    - 100,000 shares outstanding, assuming no debt
    - YTM = 5%
    - Tax Bracket = 35%
    - Interest Rate = 6%
    - There is no preferred stock
6. In the prior question, state not just the EBIT, but its consequent levels of ROE and EPS.
7. This text solves the question of the Crossover Point by reference to ROE. Present the parallel formula using EPS.
8. If debt is cheapest, why not use it to the max?
9. List and discuss the merits and demerits of using debt in a firm's capital structure.
10. How does debt interact with Operating Risk, if at all? Explain.
11. A guiding principle in determining the optimal level of debt has to do with minimizing the firm's Weighted Average Cost of Capital. Explain.
12. Another guiding principle has to do with maximizing the firm's value. Explain.
13. What four variables are relevant in determining the firm's actual degree of leverage? Explain.
14. What is meant by External Funds Needed?
15. What are Internal and External Funds?
16. Why is the notion of External Funds Needed so very important?

Selected Answers

Question 5:

It does not matter what amount of leverage you assume; the crossover point will be the same for all amounts of leverage. Let us assume 50%. Why not?

(EBIT) (.65) / 2,500 = (EBIT- 75) (.65) / 1,250

EBIT = 150

Question 6:

ROE = (150) (0.65) / 2,500 = 0.039

EPS = (150) (0.65) / 100 = $0.975

Chapter 6: Capital Structure Theory

6.1 Chapter Six Learning Outcomes

Learning Outcomes

- **Calculate** *Homemade Leverage.*
- **Explore** how Homemade Leverage affects the notion of whether a firm's capital structure "matters."
- **Connect** the notions of the Levered Beta with the firm's Return-on-Assets.
- **Consider** Modigliani and Miller's first proposition of their famous Capital Structure Theory.

6.2 "Homemade" Leverage Illustrated: An Introduction to Modigliani & Miller ("M&M") The Financing Decision

In this section, we shall focus on issues related to the firm's financing decision, i.e., the extent to which the firm relies, alternatively on debt or equity in order to finance its capital investments.

We shall now challenge the statement that debt may positively affect company value via EPS. M&M's argument shall be that leverage does not affect the shareholders' equity claim nor alter the overall firm's risk. All it does is shift risk between lenders and shareholders. We shall show that a shareholder can create his own investment leverage regardless of the investee company's leverage. Debt matters to EPS, but does it really matter to price? The following illustration is drawn from the earlier "financial leverage" illustration; here, we will use "Case B" with an EBIT of $40,000. We shall compare the two cases of no leverage versus 50/50 leverage.

1-B. Unleveraged Structure, Absent Taxes ("Case 1-B")

EBIT	$40	
Less Interest Expense	($00)	
Earnings Before Taxes	$40	
EPS (Unleveraged)	$4.00	(10,000s.)
Times Shares Owned	×100s.	
	$400	

The investor has a kind of interest (or "claim") in $400 of the firm's profits (EPS). The common stock investor "owns" the EPS, which will be either, in part, paid out as dividends, or retained. In both cases, the common stock investor has an interest in the EPS and what is done with it.

2-B. Leveraged Structure, Absent Taxes ("Case 2-B")

EBIT	$40	
Less Interest Expense	($12)	
Earnings Before Taxes	$28	
EPS (Leveraged)	$5.60	(5,000s.); (28/5 = 5.60)
Times Shares Owned	×100s.	
	$560	

1-B$_2$. Homemade Leverage, Absent Taxes

(Assume the investor now buys 200s., borrowing 100s.)

EPS (Unleveraged)	$4.00	
Margin Interest	($1.20)	(.12 × $20 × 0.5)
Net Income to Investor	$2.80	
Times Shares Owned	×200s.	
	$560	

An investor may purchase stock with 50% down and borrow – or margin – the balance. Thus, by borrowing, an investor can emulate a leveraged company, should his risk-profile so require, even though he may have chosen to invest in an un-leveraged company. (We concede the *ceteris paribus* simplification of utilizing the same borrowing rate – 12% – for both the company and the individual. Remember, however, that the individual's margin loan is collateralized thus making it less risky to the lender.)

By investing in an un-leveraged company (i.e., one with no debt) and borrowing money at, in this case, 50/50 leverage, the investor can emulate a cash investment in a leveraged company! Therefore, capital structure, given this thought, ought not matter to the firm's valuation! It can also be shown (but we will not) that the reverse, i.e., "de-leveraging," can be accomplished as well.

Follow-up Question:

Should a firm have 0% or 99.9% leverage? (We must leave at least one share to an owner.) Does it matter whether the corporation leverages its Balance Sheet or the whether the investor creates his/her own (homemade) leverage? Is firm value in any way affected, one way or the other? We again ask the question: What is the basis therefore for evaluating a firm's capital structure, its risk, and valuation?

Summary Activities:

Let's say the investor has $1 million to invest.

1. S/he can choose a company that has less leverage than she wants for him/herself, purchase $1 million dollars' worth of stock and *borrow* another million on margin, to achieve a, say, personal leverage 50/50 ratio. We did just this in "1-B2."
2. Alternatively, if s/he chooses a firm to purchase with too much leverage for his/her own risk profile, s/he can invest a portion of his/her money and *lend* the rest. This reduces his/her leverage. One *lends* simply by purchasing Treasury securities; this is equivalent to lending money to the US' Treasury / government.

6.3 Leveraging versus De-leveraging

We will use brand new numbers. Let's say an investor has $1,000,000 to invest. There are two basic possibilities given whatever their desired leverage ratio is. They can purchase a non-leveraged firm and leverage it using homemade leverage, or they can buy a leveraged stock and de-leverage it.

Leveraging:

1. The investor has $1,000,000 to invest.
2. The investor purchases $2,000,000 worth of stock.
3. S/he borrows half, or $1,000,000.
4. Therefore, their "LMV," or "long-market value" is $2,000,000 and their loan is $1,000,000.
5. S/he now has a leveraged investment with equity of $1,000,000.

De-leveraging:

1. The investor has $1,000,000 to invest.
2. S/he invests half, i.e., $500,000, in stock.
3. S/he lends half, i.e., $500,000.
    - S/he can lend simply by buying Treasuries to, in effect, be a lender – to the government.
4. Therefore, his/her "long-market (stock) value, or "LMV," is $500,000, and his/her loan asset is $500,000.
5. S/he has now de-leveraged his/her leveraged stock investment.

6.4 The "Levered" Beta

Here we shall return to and continue with the M&M (Modigliani and Miller) argument that the firm's worth is a function of its EBIT (and ROA), i.e., its ability to produce operating profits rather than how its capital is structured in one or another proportion. Thus, it does not matter how one leverages the company, as its basic earnings power (EBIT or EBITDA) drives its worth.

Stocks and Bonds may both be said to have Betas, i.e., "relative risk." The firm's overall Beta is, again, a function of the size of its EBIT (or EBITDA) and the volatility thereof. Thus, it may also be viewed simply as the weighted average of the respective stock and bond Betas, where the weights are the relative values of the Debts and Equity with respect to the firm's total assets. While mathematically the bond and equity Betas may change, the overall firm Beta does not. This is especially true if you accept M&M's theory that firm risk, and hence valuation, are not a function of the firm's capital structure and magnitude of leverage, but of its basic earnings power, EBIT and ROA, as determined by the extent and nature of its assets and the efficiency with which the assets are employed (or lack thereof).

As debts are senior to equity, the Betas of debt ("Debt Betas") tend to be relatively low, while Equity Betas are higher. There may be occasional periods of interest rate volatility (e.g., the late 1990s and 1994-5), when Debt Betas too can be significant. A firm's overall Beta is affected by the strength of its operating cash flows (EBITDA) which are not influenced by financial leverage (i.e., debt to equity ratios), but by its business and operating risks. Thus, a restructuring of the firm's debt ratios (financial leverage), due to IPOs, re-financings, or otherwise, will affect only the Betas of its stocks and bonds, but not the firm's overall riskiness, the firm's Beta. The firm's valuation will thus be unaffected by capital structural changes.

As new equity is issued, the firm's debt Beta is reduced (and vice versa) because the equity tends to buffer the debt risk. What happens to the Equity Beta as debt is reduced?

	Initial Structure	**After Debt Financing**
Debt/TA	40%	50%
Equity/TA	60%	50%
Debt Beta	0.1	0.2
Equity Beta	1.35	?
Overall Firm Beta		

Find: The firm's overall Beta (pre- and post-financing), and its new Equity Beta.

Initial: β Firm = (.4× 0.1) + (.6 ×1.35) = 0.85

Post Financing: $0.85 = (.5 \times 0.2) + (.5 \times \beta_{Equity})$

$\boldsymbol{\beta_{Equity} = 1.5}$

Issuing more debt *raises* the firm's equity Beta (β_{quity})! (The effect would be similar for using Free Cash Flow to increase debt.) This may not hold if the size of the "firm pie" increases.

Moral of the story: This does *not* prove that the firm's value is a function of its EBIT and ROA risks/volatility. It does confirm, however, that the firm's operating risk (EBIT volatility), on the one hand, and, on the other hand, the respective risks of its individual capital components (bonds and stocks) are somewhat interdependent.

6.5 Modigliani & Miller ("M & M"): "Proposition One" The Formula

In the following discussion, it is important to keep in mind the arguments already presented to this point.

Before we get further into it, let's (re-) state the underlying restrictive assumptions:

1. Markets are perfect
    - Managers and investors share identical information about the firm's future earnings prospects
2. Corporations and investors can borrow at the same rate
3. There are no taxes, no transaction costs
    - This restriction enables M&M to assert that investors can "create" their own dividend policy.
    - Investors are indifferent as between receiving dividends or the allowing the corporation to instead retain earnings

"Proposition I": "The Irrelevance Proposition"

This (first of numerous) proposition(s) states that Capital Structure does not matter. Therefore, the value of a company's assets and hence (if the balance sheet is to match) the (market) value of its capital – including both its debt and equity, is the present value of its operating earnings (EBIT) discounted a rate of return commensurate with its risk, as defined by the standard deviation of the firm's EBIT. We can say this formally as:

$$V = D + E = EBIT / R$$

Where:

V = M =	… the market value of the firm … the market value of the firm's assets
D + E =	… the market value of the firm's total capital.
EBIT =	… Operating earnings
R =	… a discount rate commensurate with the firm's operating/business risk

Some of the implications include:

1. Changes in capital structure alter the respective risks of the capital components, but not the overall firm risk or value. This is to say succinctly that a levered (or unlevered) *Beta* for any given capital

component is independent of firm (i.e., operating or unsystematic) risk. The respective Betas of the firm's capital can change with no change in overall firm Beta.

- If there are no taxes or transaction costs, stock prices and equity capital costs are also unaffected.

2. There is a significant dichotomy between the firm's business- and financial-risks.
    - EBIT, i.e., operating profit, is generated by the firm's assets (and management's skill in exploiting the assets).
    - Business risk has to do with the variability of the firms' operating cash earnings (EBITDA). Financial risk has to do with financing choices and its proportionate distribution amongst *lenders* (i.e., paying interest) and *owners* (i.e., paying dividends and retaining additional earnings).
    - Increasing leverage passes on more financial risk to shareholders.
3. The discount rate, which is used, is the required return on the firm's assets, and is based on the variability of EBIT and thus ROA as well.
4. In the absence of debt (i.e., an all-equity capitalization structure) the return on equity will equal the return on assets: ROA = ROE.
    - If there is no debt, EBIT = NI, EQ = TA, and EBIT / TA = NI / EQ.
        - Remember: we are still ignoring Taxes.
    - This also says that leverage can potentially enhance ROE (vs. ROA) similar to what we saw with the DuPont Model.
5. Although this model, as it is, does not neatly – or at all – conform to actual market conditions, M&M have nonetheless, provided us with some valuable insights.

M&M offered additional propositions, increasingly relaxing earlier assumptions and providing additional alternatives.

- M&M do not address stock pricing – unless you assume no taxes (i.e., Prop I). That's the domain of the DDM.
- M&M focus on the firm's overall risk and value, not bond or stock prices.

Again,

Business Risk = f (σ EBIT, σ ROA)

Financial Risk = f (σ NI, σ EPS, σ ROE)

6.6 M&M and Pizza

One of the great 20th-century philosophers was a former baseball player named Yogi Berra.

One day, Mr. Berra walked into his favorite pizza shop in New Jersey and ordered a whole pie for himself. The waiter asked him, "How many slices would you like me to cut the pie into – four or the standard eight?"

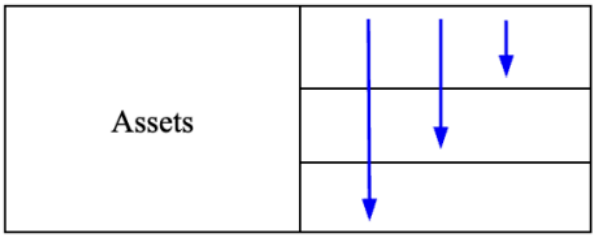
Without pausing, Mr. Berra replied, "Four. I don't think I can finish eight."

At the same time, Dr. Franco Modigliani went into his favorite kosher pizzeria in New York. He too ordered a whole pie for himself. The waiter asked him: "How many slices would you like me to cut the pie into – four or the standard eight?"

Without pausing, Dr. Modigliani replied, "It doesn't matter how you slice the pie; it's all the same."

It is unknown whether Mr. Berra and Dr. Modigliani ever met or spoke; perhaps they could have ordered one pizza to share – four slices each.

"Some Company's" Balance Sheet
as of Now



The arrows signify varying possible amounts of debt. It doesn't matter how much leverage the corporation uses, how the pie is sliced, it is assets, EBIT, and ROA that determine Corporate Value.

Every man, wherever he goes, is encompassed by a cloud of comforting convictions, which move with him like flies on a summer day.

Bertrand Russell
Skeptical Essays (1938)

6.7 Leverage After-Thought

Photo by Pixabay CC0

> *It doesn't matter whether a company is big or small. Capital structure matters. It always has and always will.*
>
> – Michael Milken, Chairman, the Milken Institute

6.8 Review Questions: Chapter Six

Review Questions: Chapter Six

1. Express the notion of “Homemade Leverage” in words.

2. How, if at all, does the theory of Homemade Leverage affect overall firm valuation?

3. An investor may use Homemade Leverage to either leverage up a low-leveraged firm, or to de-leverage a highly-leveraged firm. Explain how he may do either or both.

4. Explain the notion of the “Leveraged Beta.” In the course of doing so, reference Return-on-Assets.

5. You are given here Modigliani and Miller’s basic formula for firm valuation. Explain what each element means and from whence it comes. V = D + E = EBIT ÷ r. Why is this an equality?

6. Define separately Business- and Financial-Risks. What are the key variables to each?

7. In summary, how does a firm’s capital structure interact with its overall valuation – according to M & M? How does the leverage ratio inter-act with the valuation of each of its individual capital components?

8. What do you make of all this M & M stuff?

Chapter 7: Dividends and Capital Structure

7.1 Chapter Seven Learning Objectives

Learning Outcomes

- **Connect** the dots between a firm's cash dividend payment and its prospective capitalization.
- **Explore** numerous Dividend payment theories.
- **Relate** Stock Dividends and Stock Splits to Capital Structure
- **Distinguish** between the effects of paying a cash dividend or using the funds to buy back stock.
- **Develop**, once again, a firm's Optimal Capital Structure in the context of dividends.

7.2 The Effect of Paying a Dividend on a Firm's Prospective Capitalization

In the course of our earlier discussion of (vertical) capital structure, let's take a look at the effect of dividends on a firm's prospective structure. We will see that the choice of paying a dividend or not, will affect the firm's capital structure going forward.

This analysis is forward-looking. We will answer the question of what the prospective effect will be – on a firm's capitalization – of paying a cash dividend versus not paying any dividend. This is a complex decision the firm must make, one that has multiple effects, including, once again, the effect on the firm's capital structure and more.

We will assume that the firm has, in the most recent period, earned positive net income. Will the firm's leverage, its level of indebtedness, be increased or not as a result of paying the dividend?

	Initial	**Paying a Cash Dividend**	**Not Paying Any Dividend**	**Paying 100% Dividend**
Net Income		$100	$100	$100
Dividend to be paid		(20)	(0)	(100)
Addition to Retained Earnings		$80	$100	0
Debt	50	50	50	50
Equity	50	**130**	**150**	**50**
Debt/ Equity (Leverage) Ratio	1:1	**1: 2.6**	**1:3**	**1:1**

Clearly, if a company chooses to pay the dividend, its addition to retained earnings, and hence its capital structure will be "worse," i.e., more indebted or more leveraged, than if it chose instead not to pay any cash dividend.

Given this analysis, what are the arguments for paying a dividend or not?

7.3 Dividend Policy Theories (Prelude)

So far, we have asked numerous questions concerning capital structure (the financing decision) and related matters, and we failed to resolve them. First, we illustrated that although leverage reduces net income, it potentially enhances Earnings Per Share (EPS) and Return on Equity (ROE); we noted, in this connection that increased leverage poses a greater risk of bankruptcy and, accordingly, will reflect itself in a higher yield-to-maturity for its debt, and a higher equity discount rate, which via the Dividend Discount Model (DDM), will result in a lower share price. Implicit in all this is the notion that little or no leverage is not desirable either, as no EPS and ROE benefits will be produced.

Having satisfactorily arrived at these relationships, we nonetheless could not answer whether there is a universal solution to the notion of "optimal" leverage; the ideal level of leverage depends on industry and firm characteristics, including the stability of operating earnings. Other factors may also come into play, such as management and shareholder risk profiles.

We then posed the question as to whether leverage matters at all in light of the possibility that investors can manipulate a firm's leverage to suit their own needs by utilizing "homemade leverage," which alternatively reduces or amplifies the firm's *ex-ante* degree of leverage. An argument was made that leverage is not important at all; rather the firm's valuation is a function of the size and volatility of its operating earnings and not its Net Income (NI), EPS, or ROE. These arguments are attributable to Modigliani and Miller's ("M&M") groundbreaking research. (Their arguments were also based on very restrictive assumptions including the absence of taxes and flotation costs.)

Next, we posed the question as to whether dividends matter whatsoever – does paying, or not paying, dividends positively or negatively affect a firm's valuation? Clearly, dividends affect a firm's ability to grow its (internal) equity in that dividends paid reduce retained earnings, causing the firm to rely, in part, on more expensive external equity for growth. It would thus appear that dividends indeed matter to some extent! On the other hand, it is argued that dividends are only paid if the firm can find no acceptable capital projects, i.e., investments whose prospective returns exceed the firm's *hurdle rate*. We shall present some classic pro and con arguments concerning the subject of the "relevance" of dividends next.

In short, firms will pay cash dividends when they perceive a bright future cash flow outlook and reduce or eliminate dividends when the future is gloomy. In any case, erratic dividend payments make investors queasy, with negative stock valuation implications. If the firm needs cash to fuel growth, it may not pay any dividends. Paying dividends may indicate that the firm has no positive internal capital budgeting investment options.

7.4 Dividend Policy Theories

Are dividends *really* relevant to the value of a firm? Why pay dividends at all? For example, Warren Buffet, long-time, renowned chairman of Berkshire Hathaway has said, more or less, that he will only pay dividends when his company is unable to identify internal investment opportunities that exceed the potential return that smaller investors' respective opportunity sets present (after their paying taxes on those same dividends received). In the over five decades that he has run the firm, there has never been any dividend paid!

In terms more familiar to us, he is saying that he will pay a dividend only when internal projects' projected Internal Rates for Return (IRRs) – or some other reasonable measure – are less than what an investor may earn on his own after receiving a dividend and paying taxes on it. Remember that the opportunity set available to a large corporation, with large sums of money, is much more attractive than the set available to the typical, small investor. In other words, a firm will pay a cash dividend if it can identify no internal capital budgeting project whose return, however defined, exceeds the firm's *hurdle rate*.

The **Clientele Theory of Dividends** asserts that the firm will pay dividends if its investors are demanding it. In general, investors who want dividends, it is argued, will invest in companies that have a good dividend-paying track record.

Let's review some well-known, academic dividend policy theories. First, we are already somewhat familiar with **Modigliani & Miller's (1961) "Dividend Irrelevance" theory**.[1] As already discussed above, M&M argue that the firm's basic earnings power, that is, the operating earnings (i.e., Earnings Before Interest and Taxes or EBIT) generated from its basic lines of business, are what drive the company's valuation. The more elaborate, formal theory, which has been presented above, is quite complex, and utilizes a series of very restrictive assumptions and building blocks.

In short, M&M say that return on assets (ROA = EBIT / TA) – where TA = Total Assets – is more important than Payout Ratio (PR = D / NI), or hence Dividend (D) Growth (g). Put differently, the distribution of net income to the shareholder as between dividends and retained earnings has no effect on the firm's overall capital costs (i.e., the total Firm β is unchanged), and thus on its overall valuation. Capital structure has no bearing on a firm's value. Moreover, the capital budgeting process and investment policies are independent of its dividend policy.

Dividend policy, being irrelevant, once again, devolves necessarily from the position that capital structure does not matter; this is rationalized as investors can create homemade leverage (i.e., by ignoring, at first, the impact of – differential – tax rates, assuming equal borrowing rates as between corporations and individuals, and other restrictive assumptions.)

Gordon & Lintner (1962) espoused the **"Bird-in-the-Hand"** theory. (Myron Gordon, is the father of the "Gordon Model," which we know better as the "Dividend Discount Model.") They contend that investors prefer a dollar of dividends in the here and now over an unsure future dollar of capital gains. In a sense, *g* is riskier than D/P_0 in the DDM formula. They challenged one of M&M's assumptions, claiming that investors prefer less risky dividends, and are uncomfortable with capital gains' prospects. In contrast, M&M claimed that R_{Firm} is independent of PR and dividend policy.

A fourth theory may be referred to as "**Tax Preference**" theory. Here, an investor's choice as between a dividend

1. Merton H. Miller and Franco Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business, October 1961, pp. 411-433.

versus a capital gain depends on the investor's marginal tax bracket, and the relative dividend versus capital gains taxation consequence.

In summary, we covered four academic theories: Modigliani & Miller's (1961) "Dividend Irrelevance" theory; Gordon & Lintner (1962) espoused the "Bird-in-the-Hand" theory; and "Tax Preference" theory. Some of these notions may appear old or "exotic," but they bear consideration.

7.5 Some Further Dividend Policy Issues

1. *Information Content* – Stock prices tend to rise upon *unexpected* dividend increase announcements; alternatively, a dividend cut may be a sign of financial distress and result in a sharp price decline if unexpected. There have been instances where companies have announced huge losses, but maintained the dividends, rather than decreasing it in order to preserve cash. In doing so, management is signaling that they have high expectations for future earnings prospects. In light of this, stock prices have been known to remain steady.
2. *Clientele Effect* – Investors may be said to gravitate to companies whose capital (and operating) policies match up with their own risk profiles and financial requirements. Companies with high/low payouts attract different investors. If high dividend firms are in short supply, market forces will bid up the price; low dividend firms may find it valuation-advantageous to increase its dividend. While payout is irrelevant, the clientele effect affects value; there is a competitive market for dividends.
3. *Residual Theory* – Due to flotation costs, firms wish to minimize frequency of new issues. This argues for reducing payouts and having high internal reinvestment. Dividends may be paid from what is left over after internal needs, while maintaining its desired debt ratio. This requires projecting future needs – before having to resort to external financing. If requirements exceed projected inflows, then dividends may be reduced or eliminated.

7.6 Stock Splits

Questions:

- What *is* a "Stock Split"?
- There are Stock Splits and "Stock Dividends." What is the difference? We will discuss both.

Example of a Stock Split

We shall examine a stock split for each 100 shares of stock outstanding. Originally, we will say that the stock is worth $100 per share, and the company decides to "split" the shares on a two-for-one basis. For every share one owns, s/he will now have two shares at half the original value.

Originally: 100 s. @ $100/s.

After Split: 200 s. @ $50/s.

1. Take note that, financially, a stock split, in the short-run, is a non-event, as the total share value has not changed. The value is still $10,000 – per 200 shares. The company's total equity is unaffected. There is no change in capital structure. Debt Ratios are unaffected because there is no change in the dollar value of the firm's total equity (nor of the debt).
2. Since the shareholder now has twice as many shares as before, we might also describe this event as a 100% "stock dividend." The investor has now received more shares than s/he had before.
3. Companies will split shares if they feel the price is too high. A lower price could encourage more trading in the shares and thus more efficient price discovery.
    - At this writing (2022.04.12), Berkshire Hathaway "A" shares exceeded $523,803 per share. Warren Buffet discourages trading as his investment philosophy demands "buying and holding." One needs a lot of money to purchase just one share. High share prices may indeed discourage trading, although this case is extreme.

Example of a Stock Dividend

Technically, there is no difference between a stock split and a stock dividend. They are mathematically the same. The difference is merely a matter of degree. Recall that in the case just above, we had a 100% stock dividend, but the market will refer to it as a "2/1 split." A stock split that is equivalent to, or less than, 25% is considered a "stock dividend."

Stock Dividend Example: 5/4 stock split = "25% stock dividend"

For every 100 original shares, the investor now has 125 shares. Again, the "rule" is that up to 25%, it is called a "stock dividend." Greater than 25% is called a "stock split."

Example of a Reverse Stock Split

This is the exact reverse of our original case above, and thus requires little explanation. Now, we will have a one-for-two reverse stock split. Again, total value will not be changed. For every share owned, the investor will now have one-half a share at twice the value.

Originally: 200 s. @ $50/s.

After Reverse Split: 100 s. @ $100/s.

1. Low-priced shares, especially if in single digits, are often viewed, psychologically, as bad investments. Companies do reverse splits in order to raise share prices and thereby make the shares more credible to the public.
2. Additionally, shares trading at under $1 may be de-listed from the New York Stock Exchange.

Bottom Line: Stock Splits and Stock Dividends have NO Capital Structure implications. However, this is not so for Cash Dividends and Stock Buybacks. Let's see.

7.7 Cash Dividend vs. Stock Repurchase

If a company earns net income, it may choose to either distribute a cash dividend or buy back its own stock. (A third choice would be to retain earnings and reinvest the funds in new assets or acquisitions; we'll ignore this possibility for now.) Is there any difference, in effect, to the firm and its shareholders whether it pays a cash dividend as opposed to whether it buys back the equivalent dollar amount of stock? Let us examine this by way of example.

We are given:

The total firm equity value = $1,000,000

100,000s. of stock in the company

The book – and market – values are $10/s.

An investor owns 1,000s. or 1% of stock (1,000s. ÷ 100,000s.)

(We will ignore taxes.)

<u>After $1/s. or 10% Cash Dividend:</u>

Dividend Cost: $100,000

The new firm value = ($1,000,000) – ($1 × 100,000s.) = $900,000

The dividend reduces the firm's net worth by the amount of the dividend.

Here are the effects on the company's Balance Sheet, all else equal:

($000)

	$Equity_0$ = $1,000
Cash (Credit) = $100	Cash Offset (Debit) = (100)
	$Equity_1$ = $900

Price (Book Value) = $900,000 ÷ 100,000s. = $9/s.

An investor still owns 1,000s., or 1%, of the stock – and s/he has the cash dividend of $1/s. His/her total wealth is therefore $9 + $1 = $10/s. Absent taxes, the investor's wealth is unchanged.

- The new equity value ("V") of the firm = $900.

- The new share value = $900 / 100 s. = $9 per share (i.e., $9s.)
- The investor received $1 in dividends.
- Therefore, the investor (absent taxes on the dividends received) has $9s. + $1 in dividends = $10. This is the same as his original $10 stock investment. His wealth is unchanged.

After Equivalent 10% Share Repurchase:

Instead of paying a $100,00 cash dividend, the firm will use the same funds to buy back its own stock. The re-purchased shares will go into "Treasury Stock," which is a debit to the Equity account, thus reducing Equity.

Share Repurchase Cost: (10% × $1,000,000) = $100,000

The new firm value = $1,000,000 – $100,000 = $900,000

$100,000 worth of stock will go into "Treasury Stock," which reduces total equity. (Treasury stock is a debit balance, contra-equity account.) There is now 10% less stock, that is, 10,000 fewer shares, leaving just 90,000s. outstanding. Here are the effects on the company's Balance Sheet, all else equal:

($000)

	$Equity_0$ = $1,000
Cash (Credit) = $100	Treasury Stock (Debit) = (100)
	$Equity_1$ = $900

Price (Book Value) = $900,000 ÷ 90,000s. = $10/s. An investor now owns, as before, 1% of the company's stock.

- The new equity value ("V") of the firm = $900,000.
- Outstanding shares have been reduced by 10%. Therefore, 100,000 s. (1 – 0.10) = 90,000 shares now outstanding.
- V = $900 / 90 s. = $10 per share. (Note that the numbers were rounded off.)
- The investor's wealth has not changed. She now has 900 shares (rather than her original 1,000) at $10 *plus* she received $10 for 100 shares, which were bought back by the company.

In either case, the shareholder is equally well-off, all else equal. The truly relevant question is: who will deploy the $1 in financial resources better – the corporation or the shareholder?

In the case of a cash dividend, the shareholder, of course, receives cash. If, alternatively, there is a cash buyback, the shareholder, once again, receives cash for his stock – in this case, for 10% of his/her shares. This question is relevant in the case where the corporation may have some other use for the cash, e.g., purchase of additional productive assets, a corporate acquisition, R & D or, perhaps other possibilities.

We must again note that, in this analysis, we have ignored taxes. The firm will choose whether to pay cash dividends or buy back shares depending on shareholder preferences. Of course, the foregoing alternatives affect the firm's capital structure.

The firm will buy back shares in order to increase EPS. With the same prospective income as in prior years and fewer shares outstanding, EPS must rise.

Once the firm has repurchased a portion of its shares, this act may "signal" to shareholders that more buybacks are in the works, causing the share price to increase still more.

7.8 Optimal Capital Structure (Last Minute Thoughts)

The "Optimal Capital Structure," which is represented by its debt ratio, i.e., Debt ÷ Net worth or Debt ÷ Total Assets (i.e., Debt/NW or Debt/TA), is that which results in the lowest *possible* WACC, given certain constraints. As debt increases, so too does its cost. What factors, or constraints, affect the optimal structure?

1. Management's Risk Profile

Some managements are more or less prone to take on risk. It is a matter of personality. The matching concept provides some insight into this matter. Risk profile will affect capital structure and other things.

2. Shareholder Risk Profile

Shareholders are also individuals with varying tolerances for risk-taking. Shareholders, by one argument, will appoint management and directors who share their own risk-taking proclivities. Absent that possibility, shareholders may sell shares should the firm's risk characteristics not match up with their own. (This, of course, disagrees with the notion of "homemade leverage.")

3. Industry characteristics

Some industries have lesser or greater tolerance for risk-taking. For instance, traditionally electric utilities have had high levels of debt and aggressive debt ratios. This is because debt is required for the financing of utilities' enormous infrastructure – power plants. At the same time, utility revenues are quite stable, given its captive consumer base. Customers must purchase and pay for electricity. Competition is low. Therefore,the *times interest earned ratio* or "TIE Ratio" (EBIT ÷ Interest Expense) can be low, without seriously risking default. You will recall that the TIE Ratio notes whether there are sufficient operating earnings (EBIT) with which to pay interest on the debt.

4. Relative capital costs in the market

While debt is clearly the cheapest source of capital, relative capital costs vary over time, and companies may choose to issue, say stock, when the cost of equity is cheap. It is not unusual therefore to see lots of new stocks issuances when the stock market is high.

Further, while it is necessary for a company to issue both debt and equity at the same time in order to preserve its leverage ratios' status quo, companies virtually never issue both at once. Instead they opt to issue one or the other, followed sometime later by the capital component not issued earlier.

7.9 Review Questions: Chapter Seven

Review Questions: Chapter Seven

1. State and explain all the various Dividend Policy Theories.
2. How do Dividend- and Capital-Structure Policies interact? Use quantitative analysis.
3. Do dividends matter? If so, to what do they matter?
4. Absent taxes, what is the difference between paying a cash dividend versus using the same funds instead to buy back stock? State your reasonable assumptions. Create a numerical example.
5. As an investor, do you care about dividends? Why or why not? Utilize a Valuation Model to support your own personal view.
6. A company's shares trade at $80 per share. It announces that it will split its shares 4/1. Joe owns 1,000 shares. After the split, how many shares will he own and at what value?

Selected Answers

Answer to Question #6:

4/1 Split	Pre- Split	Post – Split
Number of Shares	1,000	4,000
Price Per Share	$80	$20
Value of the Shares	$80,000	$80,000

Chapter 8: Horizontal Capital Structure

8.1 Chapter 8 Learning Outcomes

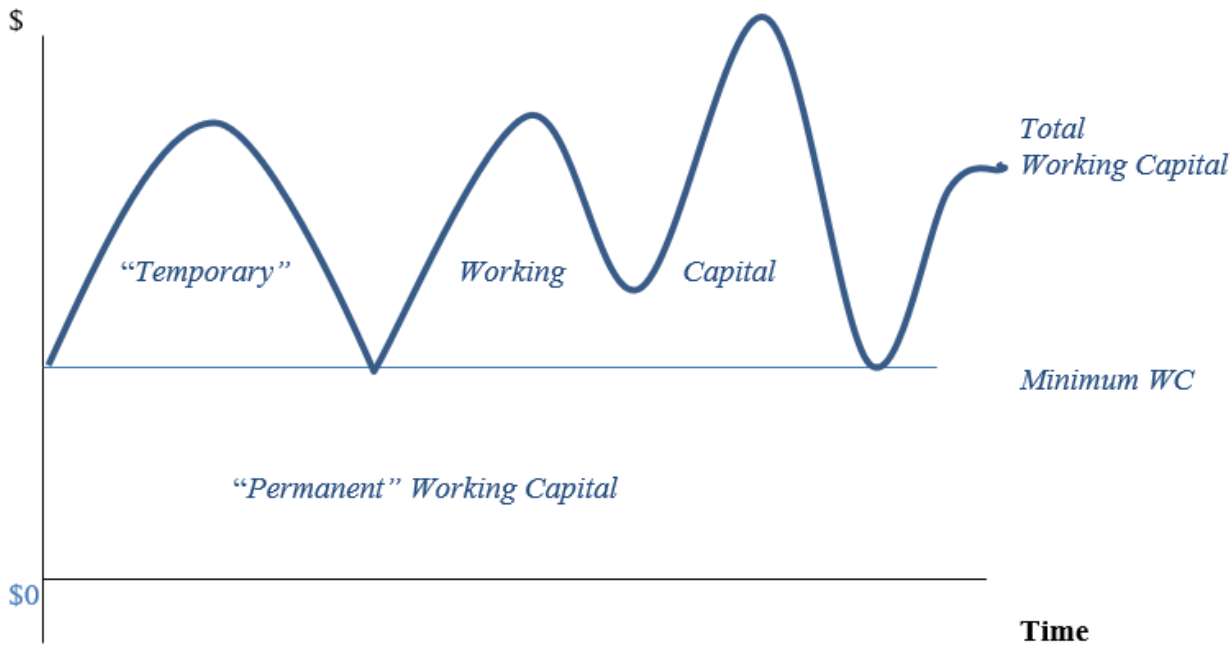
Learning Outcomes:

- **State** the difference between *Vertical* and *Horizontal Capital Theories*;
- **Distinguish** between *Temporary* and *Permanent Working Capital*;
- **Identify** the sources of both *Internal and External Funds*;
- **Develop** the *Matching Principle*.

8.2 Working Capital

Working Capital ("WC") is generally defined as Current Assets ("CA") minus Current Liabilities ("CL"). Financially speaking, and for the present purposes, "Working Capital" may be thought of as consisting of two parts: one part is "permanent," while the other may be considered "temporary." (Take note of the definitions below.)

Over time, a firm's "total" working capital may be said to fluctuate over time – for some firms more than for others. This fluctuation is pictured in the diagram below.



Permanent Working Capital (PWC) is that portion of working capital that is always present, not in a physical, but rather in a financial, or dollar, sense – when WC is at its lowest value the firm still has some funds, or resources, invested in its working capital. The firm cannot allow its cash or inventory, as examples, to go to zero. In other words, the company may be said to always (or "permanently") have a minimal amount of money invested in working capital regardless of season or business cycle effects; again, this minimal amount must be financed. This minimal amount may be referred to, oxymoronically, as "permanent" working capital. (The term "working capital" connotes short term.)

Some key relationships and definitions:

- Working Capital (WC) = Current Assets – Current Liabilities
- Temporary WC + Permanent WC = Total WC
- Current Ratio = Current Assets ÷ Current Liabilities
- Quick Ratio = (Current Assets – Inventory) ÷ Current Liabilities

8.3 Capital Financing Sources

The following table summarizes the various *sources* of funds for the corporation (and its growth). The various (right-hand side of the Balance Sheet) accounts represent various examples of the respective types of sources of funds. This list is not necessarily complete.

Funds	Accounts
Short-Term Internal Funds	Accounts Payable
Short-Term External Funds	Commercial Paper
	Bank Lines of Credit
	Short-Term Notes
Long-Term External Funds	Long-Term Bank Loans
	Long-Term Bonds
	Preferred Stock
	Common Stock
Long-Term Internal Funds	Retained Earnings

8.4 Alternative Capital Structures: Matching and Mismatching Assets and Capital Funds

Well, we have not solved the question of what is the optimal (vertical) capital structure? – and we won't! There really is no universal optimal structure, that is to say, for the ratio of debt to equity – since debt and equity are both on the same side of the Balance Sheet, we might refer to this type of capital structure as "Vertical." The actual level of leverage a company takes on will depend on numerous factors including the state of the financial markets, the company's industry and sector, and both management and shareholder risk profiles.

Some industries can tolerate, or not tolerate, a higher, or lower, interest burden given the stability, or lack thereof, in its EBIT or EBITDA; the greater the stability in its (operating) cash inflows, the greater the tolerance, objectively speaking, for leverage. For example, an electric utility company can tolerate high leverage because its cash flow is relatively stable. It can live with a lower TIE ratio. There is another way of looking into Capital Structure.

Herewith we shall discuss a related issue beyond the leverage ratio itself, that is, the internal *composition* of debt and equity relative to the firm's assets' composition, and in connection with corporate risk profiles. Let us discuss *Horizontal* Capital Structure – matching up the left and right-hand sides of the Balance Sheet.

It stands to reason that a financial manager would want to match up the timing of the firm's cash receipts and payments. When you need to make payments, you want to have funds coming in. One would think, therefore, that the firm would want to match up short-term assets with short-term financing sources (liabilities), and similarly long-term assets with long-term capital sources (debt and equity). In this way, assets would "turn over" when the liabilities come due. Funds would be available when needed. Having noted this incentive, a company, nonetheless, may have reason to "mis-match" the timing of its assets and liabilities as we shall soon see.

The following graph depicts how the corporate financial manager, who is concerned about financing sources to fund growth, may look at the Balance Sheet. Note the difference of this view in comparison to the accounting presentation. How would *you* engage the Matching Principle? How – and why – might you mismatch? Read on!

Temporary Working Capital (Assets)	Short-term Capital Funds

Permanent Working Capital (Assets)	Long-term Capital Funds, including Debt and Equity
Long-term Assets	

The manager in the table just above may be viewed as displaying a conservative risk profile. S/he would *match* short-term assets with short-term capital funds and long-term with long-term. This way, when the money is due, there will be assets that have turned over to provide liquidity. Is there an incentive for a more *aggressive* corporate financial manager to *mismatch*?

In a normal environment, long-term interest rates will be higher than short-term rates. In other words, the yield curve will be positively inclined and returns on stock will also be higher than short-term rates. Financial managers thus will have cause to finance long-term assets with short-term funds in order to reduce overall financing costs (the WACC). This, of course, increases the firm's financial risk due to faulty timing. And the actual matching decision the corporation makes emanates from the risk profile of the firm's management and thus its shareholders.

How much risk are management and shareholders inclined to take on in the name of producing greater profits, which is to say, from a non-operating, purely financial, point of view? This is the extent to which the firm will mismatch its assets and capital. This may be a viable strategy if the firm's operating cash flow is growing thereby substituting for the illiquidity of the long-term assets.

Hence, once again, as firms' managements become more aggressive, they may increasingly mismatch their capital structure (i.e., assets relative to capital) in order to lower overall financing costs. (This may or may not entail greater overall – Vertical -leverage.) The following presents some, but not all, possible structural combinations. This illustration is not intended to be a factual, or even a conceptually perfect representation; take it with a grain of salt!

8.5 Alternative Capital Structures The "Matching Principle" Illustrated

In general, companies will want to match up the duration of their assets and capital; this conservative strategy is referred to as the “matching principle” (and discussed above); this says that the (financially conservative) firm will use short-term financing sources for short-term assets, and long-term for long-term.

More aggressive managers will have an incentive increasingly to utilize short-term capital to finance long-term assets. Short-term capital is (usually) cheaper than long-term, as the yield curve normally is positively sloped. Even if the firm uses equity rather than debt, equity capital is still more expensive than long-term debt capital.

The following illustrates some possible manners in which managers may match or mis-match their firms’ assets and capital, depending on managements’ risk profiles. The chart should be taken with a grain of salt; it is not formulaic.

Here is another possible “horizontal” plan. Would you *characterize* it as conservative or *aggressive*?

Assets to be Financed	Capital Financing Source
Temporary Working Capital	Spontaneous Current Liabilities and/or Short-term External Capital
Permanent Working Capital	Long-term Debt and/or Equity
Long-term Assets	

The above plan is notably conservative. The manager has not mismatched. In mismatching, a manager runs some risks. First, there may be no cash flow when the money comes due; s/he is relying on the firm’s ability to generate sufficient operating cash flow to service the debt. That is to say, there will be no asset that will “turn over” and provide cash flow at the required time.

The manager may re-finance the short-term borrowing by taking down another short-term loan to pay off the first one – if possible. The lender may not allow this. Secondly, he runs the risk that short-term rates, at some future point, will go higher – and even exceed the former long-term rate in which case mismatching may have been a poor strategic choice.

An aggressive manager may believe that the company will grow and hence, will experience increased future cash in-flows with which short-term financing sources may be paid off.

8.6 Summary: Financial Leverage and Capital Structure

Students are very often disappointed to learn that there are no answers to the big questions. The ideal ratio of debt to equity depends on the industry, the company, its managers, and shareholders. While we may have some metrics that are indicative of one or more of the foregoing characteristics, we cannot quantify the "ideal" capital structure. We are now certainly aware of the pros and cons of too little or too much debt. The fallback notion is that the ideal capital structure is the company's current structure; otherwise, the corporation would not have chosen it. That is tough for many to buy into, especially in a dynamic world.

In the end, the best financial managers are those who are most adept at coping with uncertainty, and incomplete, or faulty, models. That's life.

How much better to acquire wisdom than gold;
To acquire understanding is preferable to silver.

-Proverbs 16:16

A wise man is strong;
a man of knowledge increases strength.

-Proverbs 24:5

8.7 Review Questions: Chapter Eight

Review Questions: Chapter Eight

1. State the difference between Vertical- and Horizontal-Capital Structure Theories.
2. How are Temporary- and Permanent-Working Capital different from one another? What is the distinguishing criterion?
3. In words, explain the Matching Principle.
4. Why would an aggressive manager mismatch his/her firm's horizontal capital position? In what manner would s/he do so?
5. Provide some examples of both Internal and External Funds.

Chapter 9: Short-Term Corporate Financial Management: Management of Cash, Inventory, and Accounts Receivable

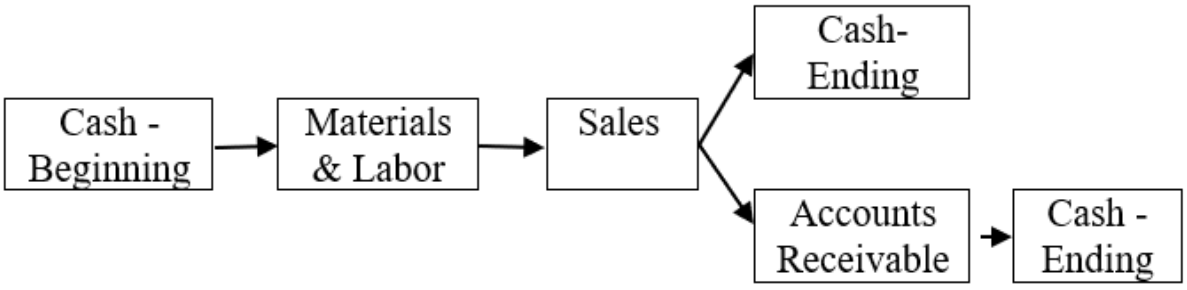
9.1 Chapter 9 Learning Outcomes

Learning Outcomes

- **Identify** the elements of the *Cash Conversion Cycle*;
- **Calculate** the *Cash Conversion Cycle*;
- **Calculate** and **Interpret** the *Cash Ratio* and the *Cash Burn Rate*;
- **Develop** a reasonable *Cash Collections and Disbursements Policy*;
- **Outline** the variables in both the *Cash and Inventory Optimization Models*;
- **Calculate** the *Optimal Cash and Inventory Quantities*;
- **Implement** an *Accounts Receivables and Credit Policy*.

9.2 The Cash Conversion Cycle

The corporation starts with cash with which it purchases raw materials (to be processed into inventory) and pays labor in order to produce goods and services for sale (see diagram below). Sales are paid for either at the point of sale or converted into accounts receivable, the latter of which are paid off later with cash. At the end of this cycle, the firm, if profitable, should have more cash than at its beginning. The Operating Cash Cycle will look as follows.

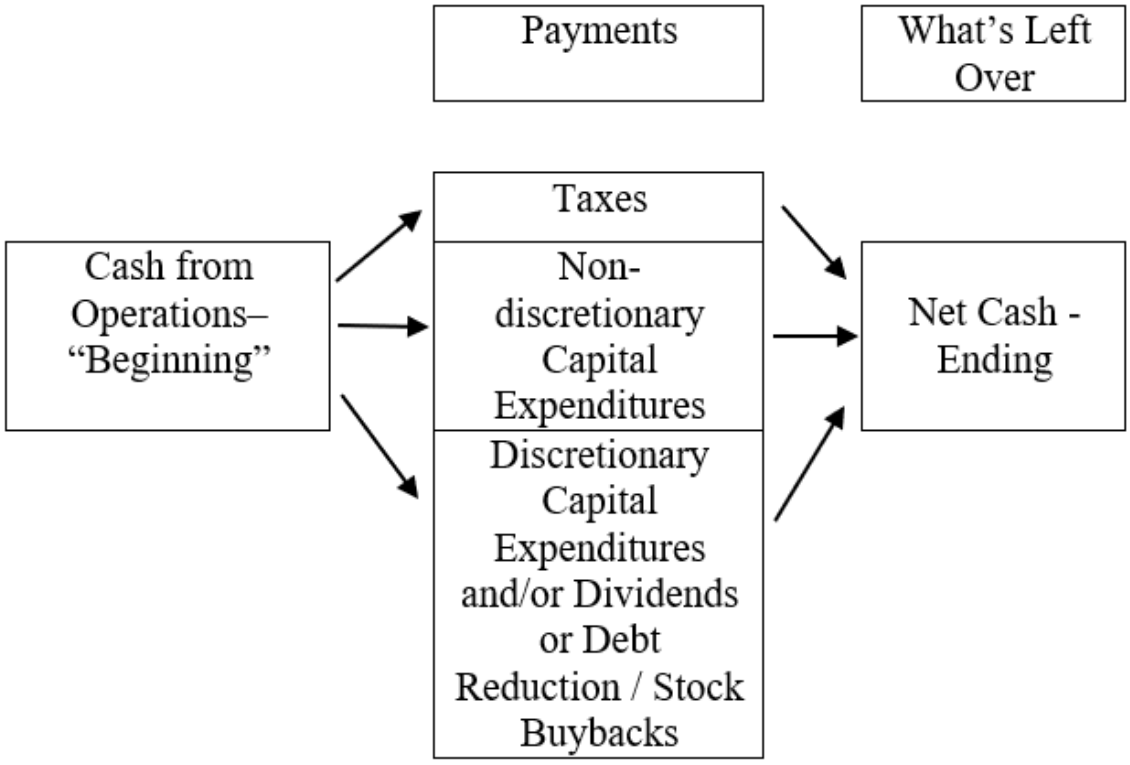


When materials are purchased, the firm, most often, will generate an "account payable," which enables the firm to keep some of its cash as a "free-loan" for up to thirty days, in most cases.

Once the cycle is completed, it starts over again, this time, hopefully, at a higher cash level. Speed is of the essence as well. The faster it can convert cash again into cash, the more rapid the employment of its assets over the course of a period, the greater its return on assets, the greater its profits, the higher its stock price!

Think of speed as a form of "operating efficiency." In this process, many firms will want to maximize sales revenues by actually charging lower prices and doing more volume, where possible, thus actually increasing sales revenues. Firms will also wish to lower operating expenses. It wants the highest productivity, in the manner of operating efficiency, from its utilization of fixed assets and manpower.

Some cash will go to taxes. Further, in order to maintain the firm's assets, some cash will be needed for non-discretionary capital- or fixed-asset expenditures. Finally, in order to fuel growth of sales, some cash will be required for discretionary capital expenditures and dividends. These final steps are illustrated below. (*Beginning Cash* below is equivalent to *Cash Ending* above.)



In summary, the cycle begins with cash and ends, if profitable, with a higher cash level.

9.3 Cash Conversion Cycle

The Cash Conversion Cycle (CCC) describes the length of time ("speed") during which the firm pays for and collects on its working capital. (Remember: Working Capital = Current Assets minus Current Liabilities or WC = CA – CL.). We will find that the firm typically has to pay for its working capital before it collects on it. A Balance Sheet is provided below. Assume Credit Sales are $20,000,000 and Cost of Goods Sold (COGS) are $4,500,000.

Sample Company

($ Thousands)

Cash and Equivalents	1,000	Accounts Payable	400
Inventories	500	Short-term Debt	6,100
Accounts Receivable	2,000	*Total Current Liabilities*	6,500
Total Current Assets	3,500	Long- Term Bonds	3,500
Property, Plant & Equip.	9,000	Common Stock	1,500
Other	1,000	Retained Earnings	2,000
Total Long-term Assets	10,000	*Total Owners' Equity*	3,500
Total Assets	13,500	*Total Liabilities + Equity*	13,500

From this information, we shall derive the Payables Payment Period, the Inventory Conversion Period, and the Average Collection Period.

Payables Payment Period

The amount of time, on average, it takes the company to pay its suppliers from time of purchase.
PPP = (Accounts Payable ÷ COGS) (360)
PPP = (400 ÷ 4,500) (360) =
32 Days

Inventory Conversion Period	**Average Collection Period**

ICP is the span of time it takes, on average, to convert raw materials into finished goods and sell it.	ACP is the amount of time it takes, on average, to collect accounts receivable from time of sale.
ICP = (Inventory ÷ COGS) (360)	ACP = (Acct. Rec'vble ÷ Credit Sales) (360)
ICP = ($500,000 ÷ $4,500,000)(360) =	ACP = (2,000 ÷ 20,000) (360) =
40 Days	**36 Days**

Some analysts will use a 360-day year, as above, based on the notion that there are twelve 30-day months to the year. We hasten to add that many analysts prefer the 365-day year. The difference is generally immaterial.

The firm pays (in this case, for its raw materials) in 32 days, but only collects on its working capital in 76 days. Let's examine this further.

Inventory Conversion Period	**Average Collection Period**
40 Days	**36 Days**
76 Days	

Again, the firm collects in 76 days altogether, but it pays in 32!

PPP	**Cash Conversion Period**
	ICP + ACP – PPP =
	40 + 36 – 32 =
32 Days	**44 Days**
76 Days	

Sample Company takes 76 days (ICP + ACP) to convert raw materials into finished goods, sell it, and collect its receivables. However, from the time it purchases the raw materials, until the firm pays for it, only 36 days pass, leaving 44 days during which time the firm must finance its current assets. The graph below illustrates, in summary, this 76-day period:

Cash Cycle with Financing Requirement

Sample Company

0	**32**	**40**	**76**
	Cash Financing Period		

Purchase Raw Materials		Convert to Final Goods and Sell	Collect receivables
ICP			
	ACP		

	Pay Accounts Payable	**Finance Payable (44 days)**
		CFP (Cash Financing Period)

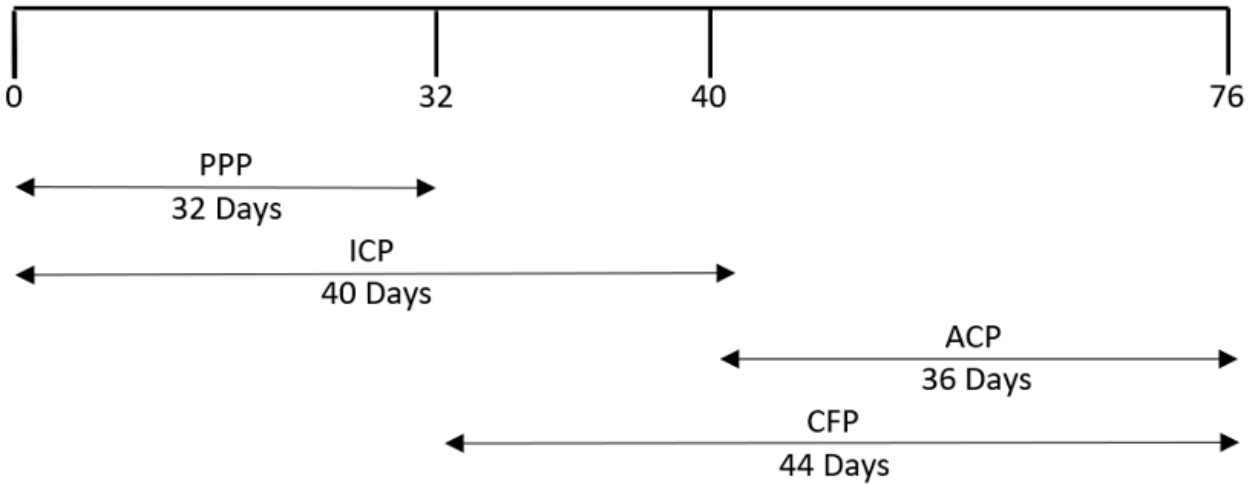
On the 32nd day, the company pays its suppliers with funds it has financed one way or another, and it pays down its financing when it collects on its receivables on the 76th day.

It must pay for the current assets with some form of short-term borrowings on which it will pay interest for an average of 44 days. If the firm can shorten its CCC (Cash Conversion Cycle), its interest expense will go down, with positive benefits to profits and share price. Can it do that?

9.4 Chronological Map of the Cash Conversion Cycle

Cash Cycle with Financing Requirement

Sample Company



9.5 Some Short-term Sources of Funds

The following are common means by which the firm will (horizontally) finance the gap in its cash flow:

Short-term Bank Line of Credit

In order to open up an "LOC," a corporation must apply for a line with its commercial bankers. Once approved, the LOC works much like a credit card. In order to borrow money, the corporations imply writes a check and funds are provided. When the corporation is able to pay back the funds borrowed, it simply writes a check back to the bank. Naturally, the firm will pay interest on the money borrowed and the bank will often demand collateral such as inventory to secure the line. LOCs are usually short-term, often up to just one year in term at which time all borrowings must be repaid. The firm may also renew the credit line.

The lending institution will often require "compensating balances." For example, let's say a firm borrows $1,000,000 and the bank requires a 10% balance. This means that the firm must always have on deposit in its account $100,000 which bears no interest. This raises the interest cost of any loans. Additionally, the bank may require an upfront fee in order to open the account. This fee is not unfair, as the bank incurs an opportunity cost in keeping sufficient liquid funds available for the customer to access.

Bank Note

A bank note although short-term, is unlike an LOC, in that the borrower receives the entire principal at the time the loan is closed and pays a fixed rate of interest until the loan matures in a year or less. The note may sometimes be collateralized.

Commercial Paper

Commercial Paper is generally an unsecured promissory note issued in the Money Market. It is not considered a Security, as stocks and bonds are, and thus is not subject to the laws governing new Security Issues (Initial Public Offerings or "IPOs") as per the Security Act of 1933. On the most reputable corporation can issue Commercial Paper because its credibility relies virtually on credit reputation alone.

In order to gain access to the money market, a corporation who needs liquid funds for as little as one day, will contact a money market dealer it knows and indicate the firm's needs. The dealer will then find buyers, or lenders, to purchase the "paper." Remarkably, these transactions are backed by trust alone. Interestingly, except for one large default in 1969, there have been scare defaults over its entire history.

The money market is an institutional market with very large minimums. Small investors can gain access to the money markets via money market savings accounts at a savings institution or via money market mutual funds.

9.6 Cash Conversion Cycle: Practice Problem

Another Sample Company, Inc.

($ Thousands)

Cash and Equivalents	1,200	Accounts Payable	650
Inventories	700	Short-term Debt	6,600
Accounts Receivable	2,200	*Total Current Liabilities*	7,250
Total Current Assets	4,100	Long-term Bonds	5,500
Property, Plant & Equip.	12,000	Common Stock	3,500
Other	2,500	Retained Earnings	2,350
Total Long-term Assets	14,500	*Total Owners' Equity*	5,850
Total Assets	18,600	*Total Liabilities + Equity*	18,600

Credit Sales = $22,450,000 COGS = $5,150,000

Solve:	**Formula**	**Calculation**	**Answer**
PPP=	(Accts. Payable ÷ COGS)(360)=		
ICP=	(Inventory ÷ COGS)(360)=		
ACP=	(Accts. Rec'v ÷ Credit Sales)(360)=		
CCC	ICP + ACP=		
CFP	ICP + ACP – PPP=		

- This company's Cash Conversion cycle is _____ days. What does this mean?
- Do you have any comments about this company? Can you draw the timeline?

Solution:	**Formula**	**Calculation**	**Answer**
PPP=	(Accts. Payable ÷ COGS)(360)=	(650/5,150)(360)=	45.4 Days
ICP=	(Inventory ÷ COGS)(360)=	(700)/5,150)(360)=	48.9
ACP=	(Accts. Rec'v ÷ Credit Sales)(360)=	(2,200)(22,450)(360)=	35.3
CCC	ICP + ACP=	48.9 + 35.3	84.2
CFP	ICP + ACP – PPP=	48.9 + 35.3 – 45.4 =	38.8

- This means that it has to obtain some short-term financing for 38.8 days. ("CFP")
- It takes 84.2 days from the time it orders raw materials (or finished inventory) until it collects on its receivables. In this timeline, it pays its Payables in 45.4 days.
- Here's the Timeline:

0	45.4	48.9	84.2
Buy Inventory	Pay A/P	Sell Inventory	Collet A/R

9.7 Cash Ratios: Firms in Financial Straits

In addition to our basic financial ratios, which we know and love, there are some further cash ratios that are relevant in the larger liquidity context. These ratios are apropos to distressed companies.

Cash Ratio = Cash ÷ Current Liabilities. (CL)

Can the firm meet its liquidity needs without relying on current asset turnover and for how long? Let's look at an example.

	20 x 1	**20 x 0**
Cash	$30,000	$35,000
Current Liabilities	$60,000	$40,000
Cash ÷ CL	**0.5**	**0.875**

In 20×1, this ratio has deteriorated from last year; without new cash, it can survive for only six months. It will need to rely more on other current assets in order to pay off its current liabilities than last year. This ratio is sometimes thought of as a "Doomsday" ratio due to its relevance to firms in financial distress. As long as its inventory turnover is alright, and it is collecting its receivables timely – with little or no bad debts, the firm's liquidity may be alright.

Cash Burn Rate = Current Assets (CA) ÷ Average Daily Operating Expenses.

Operating Expenses equals COGS plus S, G, & A (Selling, General, and Administrative expenses). This sum is then divided by 365 to get the average operating expense daily figure. The resulting Burn Rate is expressed in terms of days. This ratio tells the analyst for how long the firm can rely on its current assets to pay off its current liabilities – the longer the better; the greater the number of days the better. This ratio is especially useful in analyzing start-ups that may have scarce revenues, and therefore little ability to rely on cash and other current assets.

On the other hand, if a company has too much cash, the analyst may question why the firm's cash is not invested in productive assets. Let's look at an example of the Burn Rate.

	20 x 1	**20 x 0**
COGS	$50,000	$40,000
S,G, & A	$10,000	$8,000
Annual Operating Expenses (AOP)	$60,000	$48,000
Average Daily Expense (ADE) = AOP ÷ 365	$164.38	$131.51
Current Assets	$120,000	$110,000
Burn Rate = CA ÷ ADE	**730**	**836**

In this example, our company's *Burn Rate* has deteriorated. The company's other liquidity ratios should also be examined.

9.8 Cash Optimal Order Quantities Model: Baumol's Cash Optimization Model

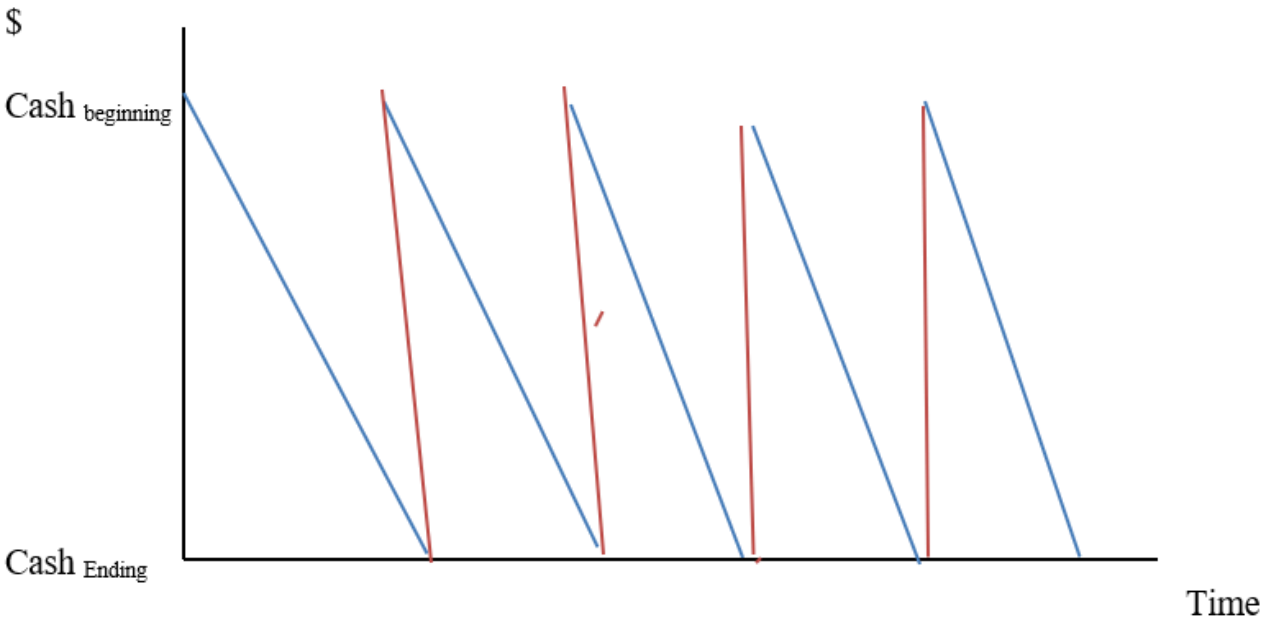
Corporations must maintain cash in order to pay salaries, accounts payable, taxes, etc. They receive cash from sales and other activities; not all cash receipts are required for immediate, outgoing payments. Some cash will therefore be invested in low-risk, short-term securities in order to earn some return on the funds, rather than having the funds sit idly in a non-interest-bearing checking account. The question of how much to hold in ready cash reserves and how much to invest, is the job of the corporation's cash manager. For corporations large enough to have a cash management department, millions of dollars are potentially being invested; naturally, the interest earned may be substantial.

The cash manager's objective is to *optimize* cash level. "Optimize" means not too much or too little. If the corporation maintains too much cash on hand, it incurs an *opportunity cost* with regard to un-invested funds; it will not have earned any interest that it should have garnered. If it does not leave enough cash on hand, it may (unnecessarily) need to resort to costly borrowing in order to meet its payments. How much cash does the firm need to hold and, accordingly, how much in ST securities does it need to periodically liquidate ("order") in order to meet its liquidity needs, e.g., for payroll, payables, etc.?

Below we shall construct a mathematical model aimed at resolving these issues; the model will be based on certain (initial) restrictive assumptions. Note that, under the assumptions (outlined below), there shall be a *saw-tooth* pattern resulting from periodically selling ST (money market) securities in order to replenish cash. The order quantity will be denoted by an undefined dollar point on the vertical axis with a diagonal line drawn to the horizontal from it. When cash reaches the horizontal axis minimum, the same quantity of cash is re-ordered and the pattern is repeated. At this point, we do not know the cash/dollar order amount on the vertical axis, nor the length of the intervals between re-order points on the horizontal.

9.9 Baumol's Model Illustrated

Here is a graph of Baumol's model. Take careful not of the assumptions stated immediately below the graph. The Blue lines represent the smooth usage of the funds over time. Thus, the interval between which the funds are replenished (red lines) are uniform – at least theoretically as per this graph.



Assumptions:

- Cash outflow is uniform, or smooth, over the period
- Length of each order period/interval is the same
- Amount ordered each time is the same
- The point of origin in the diagram need not be equal to $0. Instead, it represents the minimal point, which the firm will allow its cash to reach.
- The firm's opportunity cost is the same rate as its borrowing cost.

9.10 Cash Optimization (Baumol) Model: The Mathematics

While the firm wishes to *minimize* both its transaction and opportunity costs, it cannot do both simultaneously. In order to minimize transaction costs, it would have to infrequently order large quantities of cash; in so doing, it would increase its opportunity costs and vice versa. The relationship between transaction and opportunity costs is inverse. In the end, this shall be a mathematical problem involving the *optimization* of two inversely related functions: Transaction Costs + Holding (Opportunity) Costs. We shall formalize the foregoing notions mathematically:

Total Cost of *Average* Cash Balances = Transaction Costs + Holding (Opportunity) Costs

Next, we shall break down the formula into its components and related objectives:

1. Minimize Transaction Costs of converting short-term assets (i.e., money market instruments) to cash

   Transaction Cost = $(T/C) \times F$

   T = total amount of cash needed yearly, for all corporate business transactions

   C = amount of cash provided ("ordered") by (either) selling ST securities (or borrowing); also, total outlay required per sub-annual period; the "order quantity"

   T/C = number of transactions over the course of the year

   F = Fixed Transaction costs – for either selling a ST security or getting a loan; this excludes variable component having to do with size

2. Minimize Opportunity Cost of interest income forgone from not having invested in ST securities

   C/2 = average cash balance held during period (alternatively, beg. and end. cash balance ÷ 2)

   $(C/2) \times i$ = opportunity cost of holding this balance, where i is the interest rate foregone

3. Total Cost = Transaction + Opportunity Costs = $(T/C \times F) + (C/2 \times i)$

Given the inverse relationship of the components, we shall try to minimize total costs, i.e., to optimize cash.

9.11 The Inverse Relationship between Opportunity and Transaction Costs

Once again, transaction and opportunity costs are inversely related. This makes calculating the optimal order quantity problematic. In order to understand the inverse nature of the relationship more clearly, let's see what happens to the total cost of holding cash, given alternative cash ordering levels. Complete the table below utilizing the given data.

Given:

T = Total required yearly outlay = $25,000,000 (yearly)

F = $100

i = .05

C = variable

Total Cost = Transaction + Opportunity Costs = (T/C × F) + (C/2 × *i*)

Order Quantity (C)	Total Cost (TC)
$600,000	
$500,000	
$400,000	
$300,000	
$200,000	
$100,000	

9.12 Opportunity and Transaction Costs: (Solution)

Here is the solution (and the "givens") to the problem above.

We were given:

1. T = Total required yearly outlay = $25 million (yearly)
2. F = $100
3. *i* = .05

Total Cost = Transaction + Opportunity Costs = (T/C × F) + (C/2 × *i*)

Order Quantity	**Transaction Costs**	**Opportunity Costs**	(T/C x F) +	(C/2 x *i*) =	**Total Cost**
	(T/C x F) +	(C/2 x *i*) =			
$600,000	($25,000,000/ 600,000) (100) +	(600,000/2) (.05) =	4,167 +	15,000 =	**$19,167**
$500,000	($25,000,000/ 500,000) (100) +	(500,000/2) (.05) =	5,000 +	12,500 =	**$17,500**
$400,000	($25,000,000 / 400,000) (100) +	(400,000/2) (.05) =	6,250 +	10,000 =	**$16,250**
$300,000	($25,000,000/ 300,000) (100) +	(300,000/2) (.05) =	**8,333 +**	**7,500 =**	**$15,833**
$200,000	($25,000,000 / 200,000)(100) +	(200,000/2) (.05) =	12,500 +	5,000 =	**$17,500**
$100,000	($25,000,000 / 100,000) (100) +	(100,000/2) (.05) =	12,5000 +	2,500 =	**$27,500**

You will note that as order quantities decrease, transaction costs (T/C × F) increase, but opportunity costs (C/2 × *i*) decrease. A low point in total cost – at around an order quantity of $300,000 – is also observed.

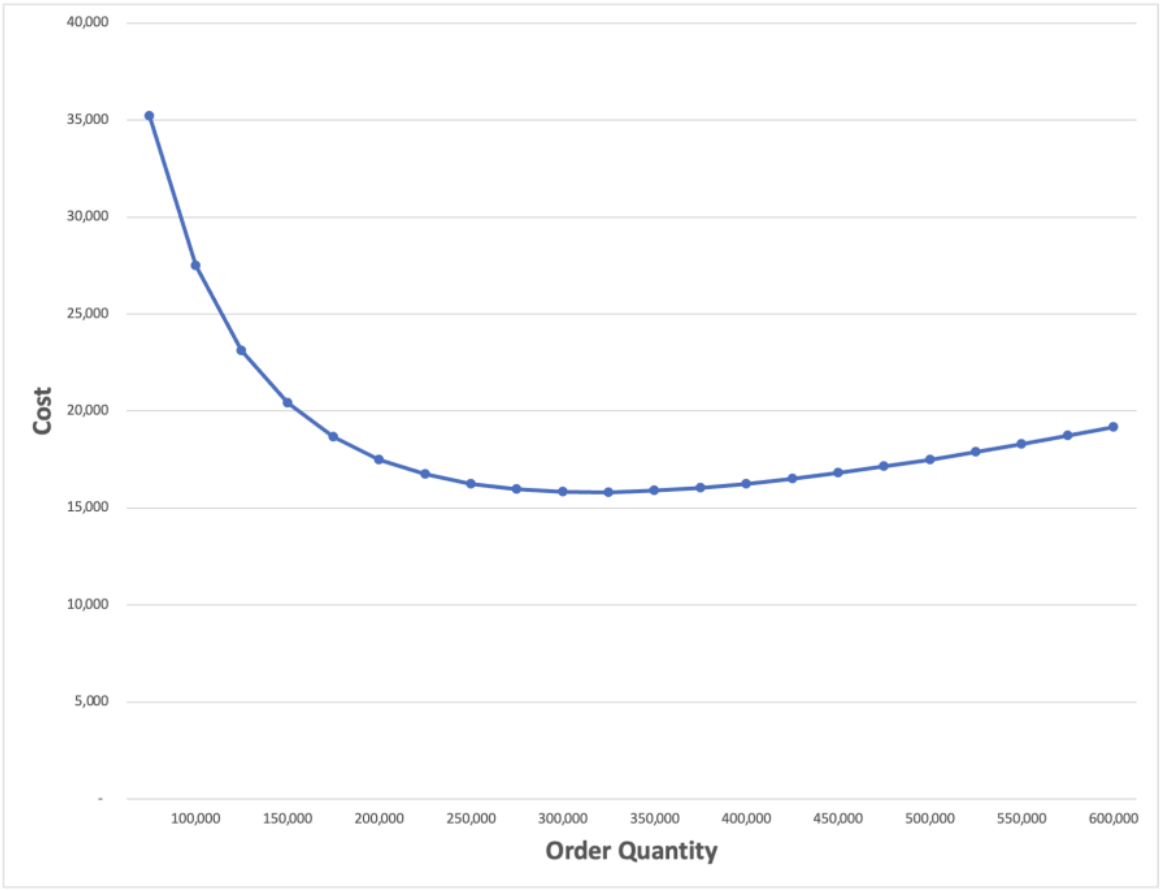
However, judging from this table alone in which order quantities were chosen arbitrarily, we cannot say that $300,000 is *the* unique solution to the problem; we can only say that it appears that the solution is in the *vicinity* of this amount (i.e., somewhere around $300,000).

We still must discover the *unique* solution to the optimal order quantity problem. For the moment, let's put this off and instead, on the next page, graph the data from the table. A graph is worth a thousand words.

It is important to note that, at around 300,000 cash ordered, the transaction and opportunity costs are approximately the same. The costs will be exactly the same at the precise optimal cash order quantity.

9.13 Illustration of the Inverse Relationship between Opportunity and Transaction Costs

The diagram below depicts the inverse relationship between Opportunity and Transaction Costs. You will note that the optimal solution which minimizes both variables appears just to the right of the $300,000 mark on the horizontal axis of the graph. Still, what is the precise and unique solution to the optimal order quantity? It would need to be calculated.



Note that as you go horizontally from right to left, transaction costs go up as order quantities are decreased, and order frequencies are increased. Also, as one goes to the left, opportunity costs decline, as ordering size decreases. The graph takes on a “U”-shape. Again, the bottom point represents the “optimal” order quantity level, i.e., the point at which both transaction and opportunity costs are simultaneously minimized. It needs to be calculated.

- This diagram has been provided by Yedidya Diena (Touro University, Lander College for Men, Class of 2024.)

9.14 The Optimal Cash Order Quantity Solution

Until now, "C" was an undefined quantity. Using some algebra (see below), we will derive C*, the optimal cash withdrawal amount, or the optimal "order quantity."

Let us see how this works in our prior example.

Given:

T = Total required yearly outlay = $25mm (yearly)

F = $100

i = .05

Due to the fact that, at the bottom of the "U" curve, Opportunity and Transaction Costs will be the same (take a look back at the table at around 300,000 units) , we may solve for C* using the formula: Transaction Costs = Opportunity Costs.

Formula and (Alternative) Solutions	**C* Proof**
(T/C) (F) = (C/2) (*i*) ($25,000,000 / C) (100) = (C/2) (0.05) $C^* =$ **$316,227**	(T/C) (F) = (C/2) (*i*) $2\ T\ F = c^2\ i$ C2 = (2 T F) ÷ i $C^* = [(2 \times T \times F) \div i]^{0.5}$
$C^* = \{(2 \times \$25{,}000{,}000 \times \$100\} \div 0.05]^{0.5} =$ **$316,227**	

Summary: At an order quantity of $316,227 (or C*), (T/C × F) = (C/2 × *i*).

Now that we know C*, the ordering interval, i.e., the number of times per year that "cash orders" must be made, also becomes known. (It is the annual cash requirement divided by C*.)

Question: Every how many days will the firm have to re-order cash?

Answer: ($25 million) ÷ (316,277) ~ 79 times a year – or about every 4 days

Notes:

1. C* will vary with F – if F is high, reduce **number** of transactions
2. The average balance will increase inversely with *i:* if *i* is high, decrease **size** of transactions
    - Therefore, if F increases, C* will increase; if *i* increases, C* will decrease
3. (This analysis assumes that market risk – in the liquidation of ST securities for meeting cash requirements, is nil.)
4. This model has also assumed a linear (i.e., smooth) reduction in cash over the period and, hence, equal intervals between re-order dates.

9.15 Cash Receipt and Disbursement Management

Sources of delays in processing checks:

- Mail Float – it simply takes some time for a check to move from payer to recipient
- Deposit Collection Float – it also takes time for a check to clear

Accelerating Cash In-flows

- Lockboxes – the recipient places a collection point in the vicinity of the debtors, thus cutting down on mailing time. Payments are then sent to this point from which the bank makes authorized collections and deposits, often several times daily. Deposit statements are transmitted to the recipient daily.
- Pre-authorized debits (PADs) – the payer provides authorization for his bank account to be debited periodically
- Wire Transfers – this can be done by telephone or by computer. The payer provides his bank's routing number ("ABA") and personal account numbers.
- Return Envelopes – this may contain a bar code, or the like, and a post office box number.
- Depository Transfer Checks (DTCs) – this is an authorization to transfer funds between accounts at the same bank.
- COD – demand "cash on delivery."

Delaying Cash Out-flows

- Delay in mail – open a bank account in a remote bank
- Centralize Payables – make one internal party responsible for payments, which would be done at the optimal time.
- Drafts – payment is made only when a draft is presented for payment, which must be examined and accepted
- Check clearing – use your experience to figure out how long it usually takes for deposited checks to clear.
- Delay Employee Payments – pay less frequently.
- In general, do not pay before the money is due.

9.16 Economic Ordering Quantity (EOQ) Model Inventory Optimal Order Quantities Model

Objective: *Optimize* inventory level. The company may experience a *saw-tooth* pattern of inventory levels. (Other assumptions shall also remain as they were in the cash model example).

As (average) inventory and order quantity *increases* so do:

- Financing costs: inventory needs to be paid from either short-term borrowings or the opportunity cost of cash invested in the short-term. (We shall, disingenuously, assume that borrowing and lending rates are the same.)
- Other carrying costs:
    - Storage & handling
    - Labor, electricity, etc.
    - Insurance
    - Perishability / (Demurrage “on the docks”)
    - Obsolescence
- To summarize: “**Carrying” costs** rise with inventory size

As inventory *increases*, the following *decreases*:

- Ordering costs – administrative
- Price/cost due to quantity discounts
- Cost of *stock-out*, i.e., not being able to fulfill customer orders
- **“Ordering” costs** decrease with inventory size

Total Cost = Carrying + Ordering Costs	
Carrying Costs = (Q / 2) (P) (C)	Q / 2 = Average inventory Carried P = Price paid per unit C = All carrying costs including financing costs (expressed as percent of cost)
Ordering Costs = (F) (S / Q)	F = Fixed cost per order S = Annual unit sales projected Q = Periodic ordering quantity (units)
(Q / 2) (P) (C) = (F) (S / Q) (Q / S) (Q / 2) = F / PC) $Q^2 = (2\ F\ S) / (P\ C)$ $Q^* = [(2\ F\ S) \div (C\ P)]^{0.5}$	

9.17 Inventory Model Mathematics Problem

This has been another optimization problem. We have derived the following formula in Chapter 9.16, "Economic Ordering Quantity (EOQ) Model Inventory Optimal Order Quantities Model" :

$$\mathbf{Q^* = [(2\ F\ S) \div (C\ P)]^{0.5}}$$

Example:

(S) Projected annual unit sales = 10,000

(C) Total carrying cost per unit of inventory = 25%

(P) Unit inventory purchase price = $5

(F) Fixed order costs = $500

Solution:

$$Q^* = [(2 \times \$500 \times 10{,}000) \div (0.25 \times \$5)]^{1/2}$$

$$= [\$10{,}000{,}000 \div \$1.25]^{1/2}$$

$$\approx 2{,}828 \text{ units}$$

Question: Every how many days will the firm have to re-order inventory?

Answer: (10,000) ÷ (2,828) ~ 3.5 times a year – or about every 102 days

9.18 Receivables and Credit Policy

The "investment" in Accounts Receivable can be measured by the opportunity cost of funds tied up in the receivables. The firm's Credit Policy is thus of great importance. (You will recall that the ACP = [(Average Accounts Receivable) ÷ Credit Sales] × 360.)

Credit Policy Considerations

Customer Creditability depends on the "five Cs," as noted below.

Character has to do with the firm's reputation. Is it a company that has paid its obligations on time in the past? Does it have a bad name?

Capacity has to do with the company's ability to pay its obligations. This has something to do with the company's earnings and cash flow. It also has to do with the company's balance sheet size and strength and, in particular, its liquidity ratios.

Capital is similar to capacity in that this too has to do with the company's balance sheet strength, especially its leverage.

Collateral addresses whether the seller is able to take back any collateral should the buyer not honor its payment obligation. Is there any "gap" between the financial obligation and the worth of the collateral?

Conditions have to do with the Macro-economy and industry status. If conditions are deteriorating, what effect may this have on the buyer's ability to pay its obligations?

The five Cs may be factored into a statistical model in order to come up with an objective assessment of the company's creditworthiness.

Sources of Credit Information

Providing a customer with credit terms of sale presents the supplier with a risk that the customer may not pay, or may pay late, thereby causing the supplier increased internal financial costs relative to the sale of goods and services. Not providing a worthy customer with credit terms may result in lost sales. Companies must carefully analyze each customer's creditability and monitor their financial situations on an ongoing basis. This is often handled within the company's accounts receivable department, often by a "credit analyst" who is dedicated to this responsibility. There are numerous means by which a supplier may inquire into a potential customer's ability to warrant credit terms, rather than requiring cash payment no later than at the time of delivery.

There are various credit reporting agencies, which gather information and supply the information to interested parties for a fee. Some of these agencies include Dun & Bradstreet ("D & B") and TRW. The reader may recognize these names as one and the same agencies that provide credit information of mortgagees and individuals who seek to finance a car purchase etc. A company's credit score (see below) may also be obtained from a credit reporting agency.

Industry associations are often good sources of information. If a company gets "burned" by a customer, the

company is likely to share that information with the trade association. The trade association, which is supported by members' dues, is charged with advancing the interests of its members. The association is thus a good receptacle and disseminator of relevant information.

Similarly, a supplier may garner information from the customer's other suppliers. Companies may be eager to assist one another under the premise that "what goes around comes around." Naturally, if a supplier is competing for the same customer, or is in any manner at odds with the inquiring company, it may be quite unwilling to share any information whatsoever; this is very often the case. Nevertheless, any experience with the customer can be invaluable.

If the customer has a banking relationship, the bank may provide some information on the company. Banks often provide checking, international trade, and loan services to companies. The credit analyst must be wary of a bank who lends money to a company, as it may be reluctant to provide negative information to a supplier, which information could jeopardize its lending facility and its ability to be repaid. Still, banks will provide some information on its accounts, even if diluted.

The credit analyst, employed at the supplier's firm, will often request an audited financial statement from the new customer. This will tell the analyst a great deal about the customer's creditability. It will also enable the supplier to gauge the extent to which, in dollar terms, credit may be granted.

In the period during which a salesman wishes to sell to a new account, the credit (or accounts receivable) department must approve the sale and set credit terms and limits. This is accomplished by gathering as much of the foregoing information as possible, and making a comprehensive assessment of the new customer's financial status and credit worthiness. As a result, sales and credit departments have a built-in conflict of interest: the credit department wants to be completely assured of payment on terms, while the sales department is eager to sell.

Consumer Credit Scoring System

Most credit scoring systems employ *Multiple Discriminant Analysis*, a statistical procedure similar to regression, which inputs data relating to a debtor's ability (and willingness) to pay down his debt. Credit worthiness is ranked or "scored." This serves to quantify one's credit rating. A minimum score would be required in order to earn credit. Some inputs, used by Sears for consumer credit, include:

- Home ownership (Use of some multiple or weight times one's TIE ratio)
- The TIE Ratio (Times Interest Earned Ratio = Earnings Before Interest and Taxes ÷ Interest Expense)
- Current job tenure
- Current debt load as a percent of annual income (X times debt ratio)
- Credit history (personal loans, credit cards, etc.)

Problems for evaluating credit scores of corporations have to do with the fact that financial ratios are often incomparable due to varying accounting conventions. This is helpful, however, as a first step. A personal credit score in excess of 700 is considered very good; a perfect score would be 850. Unfortunately, credit scoring agencies do not share with the public their proprietary formulae for arriving at credit scores.

Terms of Sale/Credit

Companies typically extend credit to their customers by allowing the customer to pay for the goods received some

time after delivery and receipt of the goods. The most common credit terms are “thirty days,” meaning that the customer agrees by accepting delivery that s/he will pay for the goods within thirty days of receipt.

Other common terms of sale include “2/10, net thirty.” This means that if the customer chooses to pay within ten days of receipt, s/he may take off 2% from the invoiced amount or, alternatively, pay the full invoice price within thirty days. For example, if the sale is for one million dollars, the customer may pay $980,000 within ten days of receipt of the goods. Whether it pays for the customer to take the discount or not is the subject of *Cost* of “*Trade Credit*” to be found below. Abuse of credit terms is clearly a risk with which the supplier must grapple; enforcement of terms of sale is one of the most difficult matters that a business confronts.

Collection Policy

Naturally, not all customers pay on time – or at all – in spite of the best efforts to gather and assess the customer’s creditability. It is therefore incumbent upon suppliers to have a policy to follow when the worst happens. How are “Past-due” accounts handled?

Many companies engage the services of an attorney or collection agency to assist in collecting on overdue accounts receivable. Short of this, telephone calls from salesmen, sales managers, or receivables managers can yield positive results. Passing along negative information to a customer’s other suppliers or trade association may or may not be fruitful. Calls to trade associations may yield helpful advice; in extreme cases, a bad debtor is punished by the entire trade who will not do business with the buyer unless it is for cash. This is a difficult situation because the industry does not want to ban the customer altogether, as doing so would cause the company to go out of business, resulting in the suppliers’ collectively (or individually) not getting paid their due.

In certain instances, a supplier will allow some customers to pay late with no consequences. Among the reasons for this liberality is the supplier’s interest in expanding market share to new accounts or entering new territories; the company may tolerate such lateness, knowing its longer-term goal of increased profits is attainable.

9.19 Altering Credit Policy

The *liberalization* of the firm's credit terms should only be done for financial reasons, i.e., if it increases (gross) profits more than ensuing costs. The effect of liberalization is that, first, sales should increase (with gross profits ensuing), but so too will bad debts and opportunity costs.

For every action there is an opposite (and not necessarily exactly offsetting) reaction. In all such decisions, management must consider the competition.

What if the firm tightens its credit policies? Here we go!

	Good Effect	Bad Effect
Raise credit Standards	• Reduction in Accounts Receivable • More cash sales (?)	• Lose some credit customers.
Decrease Net Due Period	• Speedier collections and cash flow	
Decrease Discount	• Some customers choose not to take discount	• Lose some credit customers. • Uncertain net effect on receivables.
Decrease Discount Period	• Collect sooner	

9.20 "Trade Credit": Relevant Costs

The customer sometimes must decide when to pay the supplier under the terms of sale offered. For instance, if the customer is offered terms such as "2/10, net 30," a decision must be made whether to take the discount and pay sooner, or forego the discount and pay later. We will analyze this decision by quantifying the *cost of foregoing the discount*, which is:

$$[(D\%) \div (100 - D\%)] \times [360 \div (\text{Full Credit Days} - \text{Discount Days})]$$

For example, if a company is offered 2/10, net 30, the *cost of foregoing the discount* is:

$$[(2 \div 98)] \times [(360) \div (20)] = \mathbf{0.367}$$

Essentially, the customer must consider the added $2 cost beyond the $98 base alternative, adjusted for time, hence 2/98. One way of thinking about time is to say, in this case, that there are 360/20, or 18 added 20-day periods in a year (consisting of twelve thirty-day intervals), during which , if the discount is foregone, the customer will pay an additional 2/98ths. (The foregoing assumes that purchases are made every day, hence 18 annual periods of 20 days each, i.e., 360 ÷ 20, turn over in the course of a year.)

By the way, 2/98 is the mathematical equivalent of [(100 ÷ 98) – 1]. This may help to clarify the notion.

In any event, this means that if the firm can borrow funds at a cost < .367, it should take the discount and borrow – if it needs funds to make payment.

The above formula does not consider the Time Value of Money (TVM). That is, we must account for the *compounding* of the cost over the year by employing the concept: $(1 + R)^{n}$

$$\text{Cost} = [(1.0) + (2 \div 98)]^{360/20} - 1.0 = .439$$

Of course, given the short time frame, time value of money analysis may be trivial.

Note that if a buyer "stretches" the payment beyond 30 days, the cost of foregoing the trade discount decreases, but so too does the company's creditability. For example, if payments are made in 40 days, the cost of foregoing the discount, using the first formula, becomes less:

$$[(2 \div 98)] \times [(360) \div (30)] = \mathbf{0.24}$$

In short, there is no one correct way of doing the foregoing; all the foregoing calculations must be taken with a grain of salt. Still, it is quite apparent that foregoing the discount is ill-advised.

Conclusion: It would pay for the firm to borrow $98, pay the bank interest for the 20 days, and save the nominal 2%.

9.21 Altman's Corporate Credit Scoring System

In the early 1960s, Dr. Edward Altman of NYU used "Multiple Discriminant Analysis," combining a set of 5 financial ratios to come up with the "Z-Score." This score assesses a company's probability of failure or bankruptcy, using 8 variables from a company's financial statements. The formula, including five ratios and their coefficients, follows.

It is notable that the principal ratio, i.e., the one with the largest coefficient is ROA (EBIT / TA)!

Z =

3 (EBIT / TA)

+ 1.4 (RE / TA)

+ 1.2 (WC / TA)

+ 1.0 (S / TA)

+ 0.6 (MV-Eq. / BV-Debt)

Code:

TA	Total Assets	WC	Working Capital = Current Assets less Current Liabilities
EBIT	Earnings before Interest and Taxes	Sales	Sales
RE	Retained Earnings	MV- Eq	Market Value of Firm's Equity
		BV- Debt	Book Value of Firm's Debt

Interpreting the Z Score:

Z-SCORE ABOVE 3.0 –The company is OK, according to these data.

Z-SCORE BETWEEN 2.7 and 2.99 – Alert. One should exercise caution.

Z-SCORE BETWEEN 1.8 and 2.7 – There is a high probability that the company will go bankrupt within 2 years of the date of the given financial figures.

Z-SCORE BELOW 1.80 – Probability of failure is very high.

9.22 Five Steps to Credit Management

1. Establish terms of sale
    - Payment period
    - Discounts
2. Do business on “open account” or via contract
    - Open account – obtain signed receipt only (no collateral provided nor contract)
    - Contract – promissory note, trade acceptance, or letter of credit
3. Assess credit worthiness (for open account sales terms).
    - What is your customer experience?
    - What are others’ experiences?
    - Credit agency information
    - Bank information
    - Financial statements
    - Statistical “credit scoring”
4. Establish credit limit.
    - A vendor may extend credit to customers to whom it may not otherwise do so in order to expand into a new territory or product market. This may cause an undesired increase in slow or bad collections.
    - In any event, it is incumbent upon the firm to minimize bad debts and maximize profits
    - This says that the firm should do business with new credit accounts *iff* (i.e., “if and only if”) the probability of payment times the profit exceeds the probability of default times COGS
    - Sometimes this means using judgment about future prospects
5. Collect!

9.23 Review Questions: Chapter Nine

Review Questions: Chapter Nine

1. Outline the Cash Conversion Cycle starting with the beginning Cash level.
2. You are given the following for *Huhr Company*. Calculate the Inventory Conversion Period, The Average Collection Period, and The Periodic Payment Period. (The figures are in the millions.) Assume that Credit Sales = $45,000 and COGS = $9,000.

Cash and Equivalents	2,000	Accounts Payable	800
Inventories	1,000	Short-term Debt	12,200
Accounts Receivable	4,000	*Total Current Liabilities*	13,000
Total Current Assets	7,000	Long-Term Bonds	7,000
Property, Plant & Equip.	18,000	Common Stock	3,000
Other	2,000	Retained Earnings	4,000
Total Long-term Asset	20,000	*Total Owners' Equity*	7,000
Total Assets	27,000	*Total Liabilities + Equity*	27,000

3. For how many days will the firm need to finance the gap between its payments and its cash generation?
4. How may the firm finance its gap between cash generation and required payments? What financing media are available?
5. What is the significance of the Cash-to-Current Liabilities ratio?
6. What is meant by the "Cash Burn Rate"? Why is it important?
7. Given the following, calculate both the firm's Average Daily Expenses and its Cash Burn rate.

	Year 1	Year 0
COGS	$25,000	$40,000
S, G & A	$5,000	$8,000
Annual Operating Expenses	$30,000	$48,000
Current Assets	$60,000	$55,000

8. What considerations will a firm analyze in its establishment of a credit and account receivables policy? Can you name the "Five Cs'?
9. What are the two key variables in Baumol's Cash Optimization Model? Why are they in opposition?
10. Here is Baumol's formula, without the code for the letters given. Given the data, what is the ideal cash order quantity?

$$(T/C \times F) + (C/2 \times i)$$

$$C^* = [(2 \times T \times F) \div i]^{0.5}$$

Yearly Cash Needs= $100 million	Fixed Transaction Costs = $1,000	Rate = .05

11. Outline the two variables in the Inventory Optimization Models. Why are they in opposition?
12. Here (below) is the formula for the Inventory Model, without the code for the letters given. Given the data, what is the ideal inventory order quantity?

$$(Q / 2) (P) (C) = (F) (S / Q)$$

$$Q^* = [(2 F S) \div (C P)]^{0.5}$$

Projected annual unit sales= 50,000	Carrying cost invt'y unit -= 25%
Unit inventory purchase price = $50	Fixed order costs = $1,500

13. How may a firm attempt to collect on a very late account receivable?
14. What is meant by a corporate Credit Scoring System?
15. What may be the most important variable, or financial ratio, in Altman's Credit Scoring model?
16. What are some variables that go into scoring a consumer's creditability?
17. What are the pros and cons for both liberalizing and tightening Credit Policy? Use numbers where possible.
18. A firm offers terms of "2/10, net 30." In general, would you say it would pay for the customer to take the discount or not? How come?

Chapter 10: Operating Leverage

10.1 Chapter Ten Learning Outcomes

Learning Objectives

- **Explore** how Financial and Operating are both similar and different.
- **Consider** the Operating Breakeven point.
- **Compare** competing Operating Leverage and Capital Investment plans.
- **Figure Out** the firm's Degree of Operating Leverage.

10.2 Operating Leverage

We have already studied "financial leverage," which had to do with the (income statement) relationship between operating earnings (EBIT) on the one hand, and both net income (NI) and earnings per share (EPS) on the other hand. We saw how financial leverage also impacted return on equity (ROE). In short, we observed that financial leverage increases interest expense, resulting in lower net income. However, when operating income levels are sufficiently high, financial leverage serves to magnify both EPS and ROE. When operating levels are below a certain amount – the crossover point – both EPS and ROE are worse than they would be absent financial leverage.

Operating leverage, in contrast, modifies the relationship between sales and EBIT. Operating leverage has to do with management's decision to invest – or not – in capital equipment in order to manufacture widgets, or, alternatively, to rely more on labor in the manufacturing process.

It has thus to do with the ratio of labor inputs versus capital inputs to the production process, or, put differently, fixed costs, representing capital investment, versus variable costs, representing the labor element. It is helpful to see the two types of leverage relationships in terms of the income statement.

Income Statement For Year Ending 12.31.xx		
	Sales	*Operating Leverage*
	(Cost of Goods Sold)	
	Gross Profit	
	(S,G, & A)	
Financial Leverage	**EBIT**	
	(Interest)	
	(Taxes)	
	Net Income	
	Earnings Per Share	
	Return on Equity	

We shall see that, when output is sufficiently high, it shall behoove the company to employ operating leverage, i.e., to invest in productive capital equipment – with fixed costs that are independent of the level of production, and to rely less on labor or variable costs. In such instances of high output, operating profits (EBIT) will be greater, i.e., more leveraged, than would be the case had the company invested less in capital or not at all.

We shall also discover the precise "crossover point" at which the employment of operating leverage first becomes worthwhile. In the course of this analysis, we shall not distinguish between production and sales; we shall assume that all units produced shall generate revenues – all units will be sold and the firm maintains no inventory.

10.3 Operating Break-even Point

In the analysis of operating leverage, we shall first need to establish the operating break-even point as a point of reference. In general, we break even on an operating basis, when revenues equal operating costs. At that point, EBIT, i.e., revenues minus operating costs (before interest and taxes), will be equal to zero. We may state the break-even as follows:

Revenues = Operating costs

Revenues – operating costs = 0

Further, revenue may be re-stated as unit price (P) times quantity (Q) produced and sold. Operating costs equal the variable costs per unit produced (V) times the quantity produced, plus the fixed cost component (F). We can now state our basic break-even formula as PQ = VQ + F. We can also derive some other useful formulas from this basic one, which are summarized below.

1	**PQ = VQ +F**	P=Avg. Price per unit
2	PQ – VQ – F = 0	Q = Quantity Sold/ Produced
3	Q (P – V) – F = 0	V= Variable Cost per Unit Produced
4	Q = F + (P – V)	F = Fixed Costs

You may also note that the equations above are linear. We are now ready to launch into an illustration. You are given two plans: Plans “A” and “B.” Plan B is the leveraged plan. The unit price at which the product is sold is a function of the market for the product and is independent of the leverage analysis; therefore, the price is the same under each plan.

		Plan A	**Plan B**
Price	P	$ 2 / unit	$ 2/ unit
Variable Cost	V	$ 1.50 / unit	$1.00/ unit
Fixed Cost	F	$20,000	$60,000

Variable and fixed costs are inversely related. When the company invests in fixed or capital equipment, the labor input per unit is diminished; with the use of a machine, a laborer can produce more, so the variable cost per unit is reduced. Now, let us return to the basic question: Does it pay to invest in a capital asset (or assets) or to, instead, rely on variable costs?

Note:

- Cost accountants may refer to revenues less variable cost as the “contribution margin.”
- In this analysis, we assume that all units produced are sold.

10.4 Solution to Problem: Operating Break-even Point

Step One:

Let us now turn to an essential question. What are the respective *break-even points* for each? That is, at what production level, "Q," will the firm break even under each plan, i.e., with EBIT = 0)? To resolve this, we may utilize equation #4: Q = F ÷ (P – V).

$$Q_A = \$20{,}000 \div (\$2 - \$1.5) = 40{,}000 \text{ units}$$

$$Q_B = \$60{,}000 \div (\$2 - \$1) = 60{,}000 \text{ units}$$

Construct a graph using quantity ("Q") as the horizontal and EBIT as the vertical axis. We may now graph this, as noted on the next page. The 40,000- and 60,000-unit points cut the horizontal axis, indicating that operating profits are nil.

Step Two:

The foregoing equations are linear; there are no exponents involved. So, in order to draw a graph, all we need to do is mark one more point for each Plan on the graph. On the distant end of the horizontal scale, let's take an extreme case for production of, say, 200,000 units. At that quantity, what would the operating profits be for each plan? For this, we may use a version of formula #3: Q (P – V) – F = EBIT.

$$EBIT_A = 200{,}000\ (\$2 - \$1.5) - \$20{,}000 = \$80{,}000$$

$$EBIT_B = 200{,}000\ (\$2 - \$1) - \$60{,}000 = \$140{,}000$$

We may now draw a straight line connecting the dots. You will note that there is a *crossover point*. To the right of the crossover point, operating profits (EBIT) are higher in the leveraged plan; at higher production levels, the leveraged plan ("Plan B") would produce greater operating profits than the no-leverage plan ("Plan A").

Step Three:

To close the discussion, we should need to discover the precise crossover point, i.e., the unique level of production for both plans ($Q_A = Q_B$) at which point EBIT will also be the same under both plans ($EBIT_A = EBIT_B$). If we project quantities produced to exceed the *crossover point*, it will favor the firm's use of operating leverage, i.e., to increase its fixed investment rather than employing more people. At the crossover point, both Q and EBIT will be the same for each plan, so let's use the profits formula to solve for Q. Thus, take equation #3 (Q [P – V] – F), which is the profits formula, and set the formula so that each plan's respective profits are equal to one another as follows:

$$Q_A\ (\$2 - \$1.5) - \$20{,}000 = Q_B\ (\$2 - \$1) - \$60{,}000$$
$$\text{Since } Q_A = Q_B$$
$$(.5)\ Q - (1)\ Q = -\$40{,}000$$
$$Q = 80{,}000 \text{ units}$$

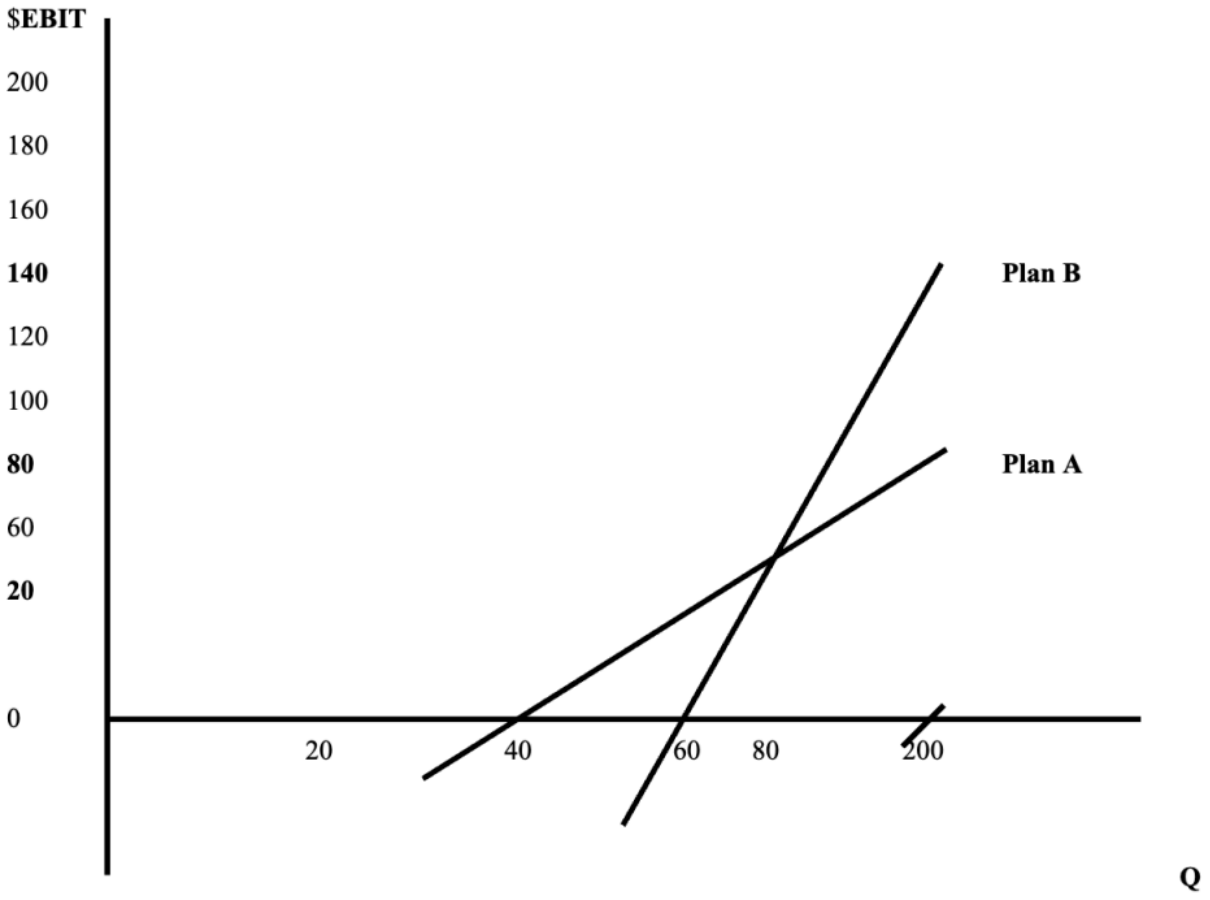
Finally, at a production level of 80,000 units, what would the operating profit/loss be under either plan?

$$80{,}000\ (\$2 - \$\ 1.5) - \$20{,}000 = 80{,}000\ (\$2 - \$1) - \$60{,}000 = \$20{,}000 \text{ of EBIT}$$

Note that it is possible for the Crossover Point to be less than the plans' Breakeven. (See end-of-chapter review question #1.)

10.5 Illustration of Comparative Operating Leverage Plans

We can now plot out the entire operating leverage picture.



You will notice that Plan B's line is steeper than A's. For every unit move in the horizontal axis (Q), there will be more movement in Plan B (EBIT) than in A – both up and down. This means there is more volatility (risk) in Plan B. This is because, once the firm's Q moves to the right of the crossover point, the fixed costs will be covered, and then EBIT will move more rapidly up and down due to the lower variable cost in Plan B versus Plan A.

In reality, even the fixed costs may not be fixed. There may come a point, at a very high production level, whereby investment in another machine may be required due to "lumpy assets." At that point, this analysis may be emulated or reiterated, considering, as if from the start, whether it pays to produce the new increment in Q with man (labor) or machine (capital).

10.6 The Degree of Operating Leverage

External analysts do not have access to the data required in order to utilize the operating Leverage formula above (PQ = VQ + F). However, studying the financial statements can yield useful inferences.

The Degree of Operating Leverage (DOL) is a measure of the sensitivity of reported EBIT to changes in reported sales. It is also a metric for operating productivity and efficiency. Operating leverage is accomplished by means of the effective use of fixed-cost assets. DOL can be measured as:

$$DOL = (\Delta \text{ EBIT}) \div (\Delta \text{ Total Sales})$$

This says that the greater the degree of operating leverage, the greater the change in EBIT, for a given change in Sales.

Let's look at an example and then see what we can make of it.

	Company A			**Company B**	
	Last Year	**This Year**		**Last Year**	**This Year**
EBIT	$1,000	$1,500		$500	$750
Sales	10,000	30,000		20,000	50,000

$$DOL_A = (1{,}500 - 1{,}000) \div (30{,}000 - 10{,}000) = 0.025$$

$$DOLB = (750 - 500) \div (50{,}000 - 20{,}000) = 0.00834$$

Company "A" has greater operating leverage. In other words, it gets relatively more incremental EBIT (2.5¢) out of every added dollar of sales than does Company "B" (0.8¢). The astute reader will recognize DOL as slope and recall that slope is a measure of risk. Sales would be on the horizontal axis, while EBIT would be on the vertical.

One need also recall that *business risk* affects the volatility of EBIT. Changes in product markets, raw materials, management, technology etc. will affect EBIT in a magnified manner both positively and negatively the greater the DOL is.

10.7 Operating Earnings (EBIT): Standard Accounting (Reporting) versus Cost Accounting

The accountant and the cost accountant each have their own different ways of arriving at the same operating earnings figure – EBIT. Let's see how each does his work.

Reporting	**Cost Accounting**
Total Revenues	Total Revenues
(Cost of Goods Sold)	(Variable Costs)
Gross Profits	*Contribution Margin*
(Selling, General, and Administrative Costs)	(Fixed Costs)
EBIT	EBIT

Neither presentation is "better" than the other necessarily; each simply serves a different function or end. The accounting report is intended to provide a summary of the financial results of the corporation for outside readers' consumption. The cost accounting analysis is intended for internal assessment and corporate planning purposes alone.

Some use Δ Contribution Margin ÷ Δ EBIT to measure DOL. Same difference in terms of ranking one company to another, but not in the actual numbers.

10.8 Liberalizing Credit Policy (A Last Look)

A company may decide to liberalize its credit policy, e.g., by extending collection terms, in order to expand into new product or geographic markets, or to increase sales in its current market. In doing so, it may have to put up with a higher percentage of late- and non-payers. The firm should do business with new credit accounts "*iff*" (i.e., "if and only if") profits increase after accounting for increased bad collections. Let's look at an example.

Current Posture (Per Unit)

Sales	5,000 u.
Price	$120
Variable Cost	$80
Fixed Cost	$15
Total Cost	$95
Terms	30 days

Due to the liberalization of Credit Policy, the company projects that its sales volume will increase by 40% and its bad debts by 5%. Assuming the firm is operating at less than full capacity, no additional fixed costs will be required.

Advantage of Policy Change	
Incremental Sales	5,000 u. × 0.40 + 2,000 u.
Incremental Unit Profit = *Price – Variable Cost*	$120 (80) $40
Total Incremental Profit = *Incremental Unit Sales* × *Incremental Unit Profit*	2,000 u. × $40 **$80,000**
Incremental Sales	2,000 u. × $120 $240,000
Increased Bad Debts	$240,000 × 0.05 **$12,000**
Net Increased Profits	**$80,000** **(12,000)** **$68,000**

This analysis ignores the *opportunity* cost of incremental resources tied up in accounts receivable.

10.9 Summary: Short-term Corporate Financial Management

In this chapter, we have discussed the essential challenges the company faces in its short-term financial management, including cash, inventory, and accounts receivables management. As before, models are helpful guidelines, but can never replace the human mind, which is far more adept at dealing with nuances and mathematical imperfections.

That is why finance people get paid big bucks! We cannot be replaced by machines and technology.

So… let's get started!

Appendix: Basic Finance Formulae

Ratios	
Current Ratio	= CA / CL
Quick Ratio	= (CA – Invt'y) / CL
Days Sales Outstanding	= (AR / Credit Sales) × 360
Inventory Turnover	= COGS / Invt'y
Debt to Net Worth	= Total Debt / Total Equity
Debt to Total Assets	= Total Debt / Total Assets
Times Interest Earned	= EBIT / Interest Expense
Gross Profit Margin	= Gross Profits / Total Sales
Operating Profit Margin	= Operating Profits / Total Sales
Pre-Tax Profit Margin	= Pre-Tax Profits / Total Sales
Net Profit Margin	= Net Profits / Total Sales
Return on Assets	= EBIT / Total Assets
Return on Equity	= Net Income / Total Equity
Sales to Fixed Asset Turnover	= Total Sales / FA
Sales to Total Asset Turnover	= Total Sales / Total Assets
Price/Earnings Ratio	= Common Share Market Price / Earnings Per Share
Dividend Yield	= Total Dividends / Common Share Market Price
	= Dividends Per Share / Earnings Per Share
Payout Ratio (PR)	= Total Dividends / Net Income
	= Dividends Per Share / Earnings Per Share

Retention Rate (RR)	= (Net Income – Total Dividends) / Net Income = 1 – PR
PR + RR	= 100%

COGS	= Beginning Inventory + Purchases – Ending Inventory

Depreciation Expense	
Straight-Line	= (Cost – Salvage Value) / Number of Years
Sum-of-the-Years' Digits	= (Cost – Salvage Value) x (Reverse Years / SOYD)
Double / Declining Balance	= (Declining Balance) x (Straight-line Rate)

Tax Shield	= (D) (T)
Free Cash Flow	= EBITDA (1 – T) + Depreciation (T) – Necessary Capital Expenditures – (Increase in Net Working Capital)
External Funds Needed	$= [(A_0/S_0)\ \Delta S] – [(AP_0/S_0)\ \Delta S] – [(M_0)\ (S_1)\ (RR_0)]$
P_0	$= (D_0)\ (1 + G) / (R – G)$
D_1	$= (D_0)\ (1 + G)$
R	$= (D_1) \div (R- G)$
R	$= (FV / PV)^{1/n} -1$
R_M	$= R_F + (R_M – R_F)\ \beta_M$
R_P	$= R_F + (R_P – R_F)\ \beta_P$
Market Risk Premium (MRP)	$= R_M – R_F$
Portfolio Risk Premium (PRP)	$= R_P – R_F$
Weighted Average Portf. Ret.	$= R_P = \sum (w_i) \times (R_i)$
Variance	$= \sigma^2 = \Sigma(x_i – x)^2 \sim (n – 1)$
Operating Break-even Profit	= PQ = VQ + F